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Dynamic Stability of a Slightly Curved Viscoelastic Pipe Conveying Fluid

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Abstract
It has been established both in theory and experiment that perfectly straight pipe is an idealisation that does not exist in practice. Viscoelastic pipes are commonly used in various industrial applications. When undergoing deformations, a viscoelastic material combines both viscous and elastic behaviours, by exhibiting time-dependent strains. A few researchers have worked on slightly curved elastic pipes conveying fluid but most of the works have not considered slightly curved viscoelastic pipe. This work analyzes the effect of the viscoelastic property on a slightly curved pipe conveying fluid. The developed nonlinear partial differential equation (PDE) of motion is decomposed and converted to a system of nonlinear ordinary differential equations (ODE) using Eigen-function expansion method. The resulting ODE is then solved by the Runge Kutta 4th order method. The dynamical analysis of the pipe is presented using bifurcation diagrams and phase plane portraits. The results obtained show that viscoelastic property attenuates buckling instability of the pipe and the route to chaos is via periodic doubling.

Keywords: *bifurcation; critical velocity; initial curvature; vibration; viscoelastic*

1.0 INTRODUCTION

Pipes used in the industries are made of different materials depending on the nature of use. They can be elastic such as stainless steel, carbon steel, copper, iron, aluminium and brass. They can also be viscoelastic such as PVC pipe. When undergoing deformations, a viscoelastic material combines both viscous and elastic behaviours, by exhibiting time-dependent strains. Two well established viscoelastic models are conventional in the literature (Ibrahim, 2010). They are Maxwell and Kelvin-Voigt models. A purely viscous damper represents Maxwell model and purely elastic spring connected in series while a Kelvin-Voigt model is represented by a purely viscous damper in parallel with a purely elastic spring as shown in Figure 1.

The advantage of the Maxwell model is that it predicts stress relaxation reasonably accurately, but is weak in predicting creep. Kelvin-Voigt model, on the other hand, predicts creep reasonably well, but weak in predicting stress relaxation. These models have been widely used for predicting the vibration of viscoelastic fluid conveying pipes. In many publications on viscoelastic fluid conveying pipes, the effect of viscoelastic dissipation is modelled as Kelvin-Voigt type (Païdoussis and Issid, 1974, Holmes, 1977, Chen and Yang, 2005, Wang et al., 2012, Özhan and Pakdemirli, 2013, Feng Liang et al.,2018).

(a) Maxwell model (b) Kelvin-Voigt model

Figure 1. Viscoelastic models

NOMENCLATURE

b = *Initial curvature amplitude*

$()' = \frac{\partial}{\partial x}$ = *Differentiation with respect to x*

$\dot{()} = \frac{\partial}{\partial t}$ = *Differentiation with respect to time*

$\overline{()}$ = *Dimensioned parameter*

z_0 = *Initial curvature*

ε = *Strain*

w = *Transverse displacement (m)*

EA = *Axial rigidity (N)*

α = *coefficient of thermal expansion (K^{-1})*

A = *Cross sectional area (m^2)*

EI = *Flexural Rigidity (Nm^2)*

m_f = *Mass per unit length of the fluid (Kg/m)*

m_p = *Mass per unit length of the pipe (Kg/m)*

β = *Mass Ratio*

P = *Pressure (N/m^2)*

θ = *Temperature (K)*

T = *Tension (N)*

ψ = *Viscoelastic coefficient*

η = *Viscoelastic damper coefficient*

σ = *Stress*

E = *Young modulus (GPa)*

L = *Pipe Length (m)*

v = *Fluid Velocity (m/s)*

Qiao et al. (2000) solved the dynamics of viscoelastic fluid conveying pipes using the Kelvin-Voigt models. They reported some unstable phenomenon such as peaks and jumps in the pipes amplitude-frequency curves. These were very sensitive to parameters such as mass ratio, flow velocities and viscoelastic coefficient. Feng *et al*. (2004) also used the Kelvin-Voigt model and found out that when the time delay is less than 10-5, that viscoelastic pipe with both ends simply supported can be considered as an elastic fluid conveying pipe. Yang *et al*. (2007) considered the effect of pulsating flow on the stability of viscoelastic pipes for the simply supported case, using the Kelvin - Voigt model. They presented stability diagrams for the effects of mass ratio and viscosity coefficient using sub-harmonic and combination resonances. Özhan and Pakdemirli (2013) modelled viscoelastic behaviour with Kelvin-Voigt type and found that viscoelasticity reduces the natural frequencies. That is, by increasing the viscoelasticity coefficient between 0.001 and 0.05, natural frequencies of the pipe decreases for transport velocity. Viscoelasticity coefficient of zero refers to an elastic Euler-Bernoulli pipe.

The dynamics of viscoelastic fluid conveying pipe was also modelled as Maxwell type by Zhao *et al*. (2001) and Wang *et al*. (2002). They found that the critical flow velocities of divergence instability of Maxwell viscoelastic fluid conveying pipes with both ends simply supported decreases with the decrease of relaxation time; while the critical flow velocities of coupled-mode flutter increases with the decrease of relaxation time. For long relaxation times, the behaviour of viscoelastic pipes is similar to that of elastic pipes. Several researchers have worked on slightly curved fluid conveying pipe. Some of the analysis include the chaos and bifurcation (Sinir, 2010), other forms of geometric imperfections (Wang 2012), conveying pulsating flows (Li and Yang, 2017), resting on linear and nonlinear elastic foundation (Owoseeni *et al*., 2018) and most recently Orolu et al (2019) on the cusp bifurcation of the slightly curved pipe. In all these works, the pipe considered is the purely elastic pipes except for Wang *et al*. (2012) who considered a viscoelastic pipe but failed to examine the effect of the material property on the dynamics of the pipe.

In this study, the slightly curved viscoelastic pipe is modelled as a Kelvin-Voigt type. The effects of the viscoelasticity on the dynamics of the pipe are investigated. Bifurcation diagrams and phase portraits are shown with periodic and chaotic motions.

2.0 METHODOLOGY

2.1 Problem Formulation

The system being considered is a pipe with initial slight curvature simply supported at both ends as shown in Figure 1. The pipe is of mass per unit length m_p, and of length, L made of viscoelastic material of coefficient η conveying incompressible fluid of mass per unit length m_f flowing in one dimensional fully developed plug flow with constant velocity $\bar{v}$.

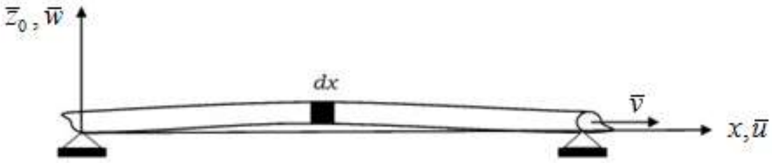


Figure 2: A simply supported slightly curved pipe conveying fluid (Orolu *et al.*, 2019)

The viscoelastic property is assumed to be of Kelvin-Voigt type. The stress can be expressed as

$$\bar{\sigma}(\bar{t}) = E\varepsilon(\bar{t}) + \eta\frac{d\varepsilon}{dt} \tag{1}$$

Where, ε is the strain, *E* is Young's modulus and η is the coefficient of viscoelastic damper. Hence, the relationship between stress and strain can be given as

$$E \rightarrow E\left(1+\frac{\eta}{E}\frac{\partial}{\partial t}\right) \tag{2}$$

Using equation (2) to modify the Modulus of Elasticity in the transverse vibration equation of slightly curved pipe conveying hot pressurised fluid as derived by Orolu et *al* (2019), the equation of motion for a slightly curved viscoelastic pipe can be written as:

$$\begin{aligned}
&\left(m_p+m_f\right)\ddot{\bar{w}}+m_f\left(2\bar{v}\dot{\bar{w}}'+\bar{v}^2\bar{w}''\right)+EI\left(\bar{w}^{iv}+\frac{\eta}{E}\dot{\bar{w}}^{iv}\right)\\
&-EA\left(\frac{3}{2}\bar{w}'^2\left(\bar{w}''+\bar{z}_0''\right)+3\bar{w}'\bar{w}''\bar{z}_0'+\bar{z}_0'^2\bar{w}''+2\bar{z}_0'\bar{z}_0''\bar{w}'-\alpha\bar{\theta}\left(\bar{w}''+\bar{z}_0''\right)\right)-\left(\bar{T}-\tilde{P}A\right)\left(\bar{w}''+\bar{z}_0''\right)\\
&-EA\frac{\eta}{E}\left(\frac{3}{2}\bar{w}'^2\dot{\bar{w}}''+3\bar{w}'\dot{\bar{w}}'\left(\bar{w}''+\bar{z}_0''\right)+3\bar{w}'\dot{\bar{w}}''\bar{z}_0'+3\dot{\bar{w}}'\bar{w}''\bar{z}_0'+\bar{z}_0'^2\dot{\bar{w}}''+2\bar{z}_0'\bar{z}_0''\dot{\bar{w}}'-\alpha\bar{\theta}\dot{\bar{w}}''\right)=0
\end{aligned} \tag{3}$$

Considering a case of a simply supported pipe, the boundary conditions are:

$$\bar{w}(0,\bar{t}) = \bar{w}(L,\bar{t}) = \bar{w}''(0,\bar{t}) = \bar{w}''(L,\bar{t}) = 0 \tag{4}$$

The initial conditions are:

$$\bar{w}(\bar{x},0) = \bar{z}_0 \tag{5a}$$

$$\dot{\bar{w}}(\bar{x},0) = 0 \tag{5b}$$

2.2 Dimensionless Quantities

The following quantities are used to transform the equations to dimensionless form.

$$w=\frac{\bar{w}}{r},\quad x=\frac{\bar{x}}{L},\quad z_0=\frac{\bar{z}_0}{r},\quad t=\frac{\bar{t}}{L^2}\sqrt{\frac{EI}{m_p+m_f}},\quad v=\bar{v}L\sqrt{\frac{m_f}{EI}}$$
$$T=\frac{\bar{T}L^2}{EI},\quad P=\frac{\tilde{P}AL^2}{EI},\quad \theta=\frac{\bar{\theta}\alpha EAL^2}{EI} \tag{6a}$$

The following dimensionless parameters are defined as

$$\beta = \frac{m_f}{m_p + m_f}; \qquad \psi = \frac{\eta}{EL^2}\sqrt{\frac{EI}{(m_p + m_f)}} \tag{6b}$$

Substituting the dimensionless quantities defined in equations (5) and (6) in equation (3), the transverse equation becomes

$$\begin{aligned} &\ddot{w} + 2v\sqrt{\beta}\dot{w}' + \left(v^2 - T + P + \theta\right)w'' + w^{iv} \\ &-\left(\frac{3}{2}w'^2\left(w'' + z_0''\right) + 3w'w''z_0' + z_0'^2 w'' + 2z_0'z_0''w'\right) - \left(T - P - \theta\right)z_0'' \\ &+\psi\left(\dot{w}^{iv} - \frac{3}{2}w'^2\dot{w}'' - z_0'^2\dot{w}'' - 3w'\dot{w}''z_0' - 3\dot{w}'w''z_0' - 2z_0'z_0''\dot{w}' + \theta\dot{w}'' - 3\dot{w}'w'\left(w'' + z_0''\right)\right) = 0 \end{aligned} \tag{7}$$

The corresponding boundary conditions for simply supported pipe are

$$w(0,t) = w(1,t) = w''(0,t) = w''(1,t) = 0 \tag{8}$$

2.3 Method of Solution

Using Eigen-function expansion method with the transverse displacement $w(x,t)$ expressed as

$$w(x,t) = \sum_{n=1}^{N} q_n(t)Y_n(x) \tag{9}$$

The corresponding normalised Eigenfunction of the solid beam is given as

$$Y_n(x) = \sqrt{2}\sin n\pi x \tag{10}$$

Using the orthogonality condition, and integrating over the pipe's length, it can be shown that

$$\int_0^1 Y_i Y_j dx \qquad \begin{cases} 0, i \neq j \\ 1, i = j \end{cases} \tag{11}$$

Considering the initial curvature as a sinusoidal function of the spatial coordinates of amplitude b, the initial curvature that satisfies the simply supported boundary condition can be expressed as

$$z_0 = b\sin \pi x \tag{12}$$

In this work, the analysis for cases up to N=10 was tested for convergence. The results beyond N=4 did not yield significant improvement on the accuracy of the critical velocity. Hence, the first four modes are considered sufficient as earlier established by Paidoussis (2014). Thus, the ordinary differential evolution equations arising from equation (7) for the first four modes are in equations (13)-(16).

$$\begin{aligned}
&\ddot{q}_1+\psi\pi^4\dot{q}_1-\frac{16}{3}v\sqrt{\beta}\dot{q}_2-\frac{32}{15}v\sqrt{\beta}\dot{q}_4+\frac{3}{4}\pi^4q_1^3+\frac{9}{4}\pi^4\left(q_3+\psi\dot{q}_1+\dot{q}_3+\frac{\sqrt{2}}{2}\pi^4b\right)q_1^2+6\pi^4q_1q_2^2+12\pi^4\left(\psi\dot{q}_2+\psi\dot{q}_4+q_4\right)q_1q_2\\
&+\frac{27}{2}\pi^4q_1q_3^2+9\pi^4\left(\frac{1}{2}\psi\dot{q}_1+3\psi\dot{q}_3+\frac{\sqrt{2}}{4}b\right)q_1q_3+24\pi^4q_1q_4^2+12\psi\pi^4\left(\dot{q}_2+4\dot{q}_4\right)q_1q_4+\begin{pmatrix}\frac{3}{4}\pi^4b^2-\left(v^2-T+P+\theta\right)\pi^2+\pi^4\\+\frac{9\sqrt{2}}{4}\psi\pi^4b\left(\dot{q}_1+\dot{q}_3\right)\end{pmatrix}q_1\\
&+9\pi^4q_2^2q_3+\left(3\psi\pi^4\left(2\dot{q}_1+3\dot{q}_3\right)+3\sqrt{2}\pi^4b\right)q_2^2+36\pi^4q_2q_3q_4+18\psi\pi^4\left(\dot{q}_2+2\dot{q}_4\right)q_2q_3+12\pi^4\left(\psi\dot{q}_1+3\psi\dot{q}_3+\frac{\sqrt{2}}{2}b\right)q_2q_4\\
&+6\sqrt{2}\pi^4b\psi\left(\dot{q}_2+\dot{q}_4\right)q_2+\frac{27}{2}\pi^4\left(\psi\dot{q}_1+b\frac{\sqrt{2}}{2}\right)q_3^2+36\psi\pi^4\dot{q}_2q_3q_4+\left(\frac{3}{4}\pi^4b^2+\frac{9\sqrt{2}}{4}\psi\pi^4b\left(\dot{q}_1+6\psi\dot{q}_3\right)\right)q_3\\
&+\left(24\psi\pi^4\dot{q}_1+12\sqrt{2}\pi^4b\right)q_4^2+6\sqrt{2}\psi\pi^4b\left(\dot{q}_2+4\dot{q}_4\right)q_4+\frac{\sqrt{2}}{2}\pi^2b\left(T-P-\theta\right)-\psi\pi^2\theta\dot{q}_1+\frac{3}{4}\psi\pi^4b^2\left(\dot{q}_1+\dot{q}_3\right)=0
\end{aligned}\tag{13}$$

$$\begin{aligned}
&\ddot{q}_2+\frac{16}{3}v\sqrt{\beta}\dot{q}_1-\frac{48}{5}v\sqrt{\beta}\dot{q}_3+6\pi^4\left(q_2+q_4+\psi\dot{q}_2+\psi\dot{q}_4\right)q_1^2+6\pi^4\left(3q_3+2\psi\dot{q}_1+3\psi\dot{q}_3+b\sqrt{2}\right)q_1q_2\\
&+18\pi^4\left(2q_4+\psi\dot{q}_2+2\psi\dot{q}_4\right)q_1q_3+6\pi^4\left(2\psi\dot{q}_1+6\psi\dot{q}_3+b\sqrt{2}\right)q_1q_4+6\sqrt{2}\psi\pi^4b\left(\dot{q}_2+\dot{q}_4\right)q_1+12\pi^4q_2^3\\
&+36\psi\pi^4\dot{q}_2q_2^2+54\psi\pi^4q_2q_3^2+9\pi^4\left(2\psi\dot{q}_1+12\psi\dot{q}_3+b\sqrt{2}\right)q_2q_3+96\pi^4q_2q_4^2+192\psi\pi^4\dot{q}_4q_2q_4\\
&+4\pi^2\left(T-P-\theta+4\pi^2+\frac{1}{2}\pi^2b^2-v^2\right)q_2+3\sqrt{2}\psi\pi^4b\left(2\dot{q}_1+3\dot{q}_3\right)q_2+54\pi^4\left(q_4+\psi\dot{q}_2+\psi\dot{q}_4\right)q_3^2\\
&+18\pi^4\left(2\psi\dot{q}_1+6\psi\dot{q}_3+b\sqrt{2}\right)q_3q_4+9\sqrt{2}\psi\pi^4b\left(\dot{q}_2+2\dot{q}_4\right)q_3+96\psi\pi^4\dot{q}_2q_4^2+2\pi^4b^2q_4\\
&+6\sqrt{2}\psi\pi^4b\left(\dot{q}_1+3\dot{q}_3\right)q_4+16\psi\pi^4\dot{q}_2-2\psi\pi^2\left(2\theta-\pi^2b^2\right)\dot{q}_2+2\psi\pi^4b^2\dot{q}_4\quad=\quad0
\end{aligned}\tag{14}$$

$$\begin{aligned}
&\ddot{q}_3+\frac{48}{5}v\sqrt{\beta}\dot{q}_2-\frac{96}{7}v\sqrt{\beta}\dot{q}_4+\frac{3\pi^4}{4}q_1^3+\frac{27\pi^4}{2}q_1^2q_3+\frac{9\pi^4}{4}\left(\psi\dot{q}_1+6\psi\dot{q}_3+\frac{\sqrt{2}}{2}b\right)q_1^2+9\pi^4q_1q_2^2+\frac{3}{4}\psi\pi^4b^2\dot{q}_1\\
&+18\pi^4\left(2q_4+\psi\dot{q}_2+2\psi\dot{q}_4\right)q_1q_2+27\pi^4\left(\psi\dot{q}_1+b\frac{\sqrt{2}}{2}\right)q_1q_3+36\pi^4\psi\dot{q}_2q_1q_4+\frac{3\pi^4}{4}b\left(b+3\sqrt{2}\psi\dot{q}_1+18\sqrt{2}\psi\dot{q}_3\right)q_1\\
&+9\pi^4\left(6q_3+\psi\dot{q}_1+6\psi\dot{q}_3+\frac{\sqrt{2}}{2}b\right)q_2^2+108\pi^4\left(q_4+\psi\dot{q}_2+\psi q_4\right)q_2q_3+18\pi^4\left(2\psi\dot{q}_1+6\psi\dot{q}_3+b\sqrt{2}\right)q_2q_4\\
&+9\sqrt{2}b\psi\pi^4\left(\dot{q}_2+2\dot{q}_4+b\right)q_2+\frac{243}{4}\pi^4q_3^3+\frac{729}{4}\pi^4\psi\dot{q}_3q_3^2+216\pi^4\left(q_3+\psi\dot{q}_3\right)q_4^2+108\psi\pi^4\left(\dot{q}_2+4\dot{q}_4\right)q_3q_4\\
&+9\pi^2\left(T-P-\theta+9\pi^2+\frac{1}{2}\pi^2b^2-v^2\right)q_3+\frac{27\sqrt{2}}{2}\psi\pi^4b\dot{q}_1+18\sqrt{2}\pi^4\psi b\dot{q}_2q_4+81\psi\pi^4\dot{q}_3-9\psi\pi^2\left(\theta-\frac{\pi^2b^2}{2}\right)\dot{q}_3\quad=\quad0
\end{aligned}\tag{15}$$

$$\ddot{q}_4 + \frac{32}{15}v\sqrt{\beta}\dot{q}_1 + \frac{96}{7}v\sqrt{\beta}\dot{q}_3 + 6\pi^4\left(q_2 + 4q_4\right)q_1^2 + 16\pi^2\left(T - P - \theta + 16\pi^2 + \frac{1}{2}\pi^2 b^2 - v^2\right)q_4$$
$$+6\psi\pi^4\left(\dot{q}_2 + 4\dot{q}_4\right)q_1^2 + 12\pi^4\left(3q_3 + \psi\dot{q}_1 + 3\psi\dot{q}_3 + \frac{\sqrt{2}}{2}b\right)q_1q_2 + 36\pi^4\psi\dot{q}_2q_1q_3 + 24\pi^4\left(2\psi\dot{q}_1 + b\sqrt{2}\right)q_1q_4$$
$$+6\sqrt{2}b\psi\pi^4\left(\dot{q}_2 + 4\dot{q}_4\right)q_1 + 96\pi^4\left(q_4 + \psi\dot{q}_4\right)q_2^2 + 54\pi^4 q_2 q_3^2 + 2\psi\pi^4 b^2\dot{q}_2 + 36\pi^4\left(\psi\dot{q}_1 + 3\psi\dot{q}_3 + \frac{\sqrt{2}}{2}b\right)q_2q_3$$
$$+192\pi^4\psi\dot{q}_2q_2q_4 + 6\sqrt{2}b\psi\pi^4\left(\dot{q}_1 + 3\dot{q}_3\right)q_2 + 2\pi^4 b^2 q_2 + 54\pi^4\left(4q_4 + \dot{q}_2 + \psi\dot{q}_4\right)q_3^2 + 432\pi^4\psi\dot{q}_3q_3q_4$$
$$+18\sqrt{2}b\psi\pi^4\dot{q}_2q_3 + 192\pi^4 q_4^3 + 576\pi^4\psi\dot{q}_4q_4^2 + 24\sqrt{2}b\psi\pi^4\dot{q}_1q_4 + 16\psi\pi^2\left(16\pi^2 - \theta + \frac{1}{2}\pi^2 b^2\right)\dot{q}_4 = 0 \quad (16)$$

Equations (13 – 16) are solved simultaneously using the Runge-Kutta 4th order method with initial conditions as $q_i(0) = 0.001$; $\dot{q}_i(0) = 0$.

3.0 RESULTS AND DISCUSSION

In the Kelvin–Voigt model, a spring and a dashpot in parallel effectively model the restoring force. At the critical velocity, the dashpot does not contribute to the restoring force, and divergence occurs as if there were no dissipation. The bifurcation diagrams showing the effects of viscoelasticity on the stability of the pipe are thereby presented.

3.1 Effect of Viscoelasticity on the Pipe Material

Viscoelasticity coefficient of zero i.e. $\psi = 0$ refers to a pipe without viscoelastic damper. From Figure 3a, the behaviour of the pipe is such that it transitions from stable to unstable position and eventually becomes chaotic. The addition of viscous damping into the pipe material improves the stability of the pipe, particularly in the post-buckling region. From Figures 3(b-d), the chaotic behaviour thins out as viscoelastic coefficient increases. Considering an extract of Figure 3a from v= 3.0 as shown in Figure 4, the route to chaos can be seen. The route to chaos is the process by which the vibrating pipe becomes chaotic as the velocity is varied. From Figure 4, the route follows a similar orderly pattern which can be termed as periodic doubling. Furthermore, the dynamic responses of the system at fluid velocity v=2.2 and v=4 representing the pre buckling and post-buckling region respectively were analysed for varying ψ using the phase plane portraits in Figure 5. The pre buckling region of the viscoelastic pipe converges to a centre while the post-buckling region tends to a stable spiral as the viscoelastic coefficient increases. Hence, viscoelasticity dampens pipe vibration and attenuates instability of the pipe.

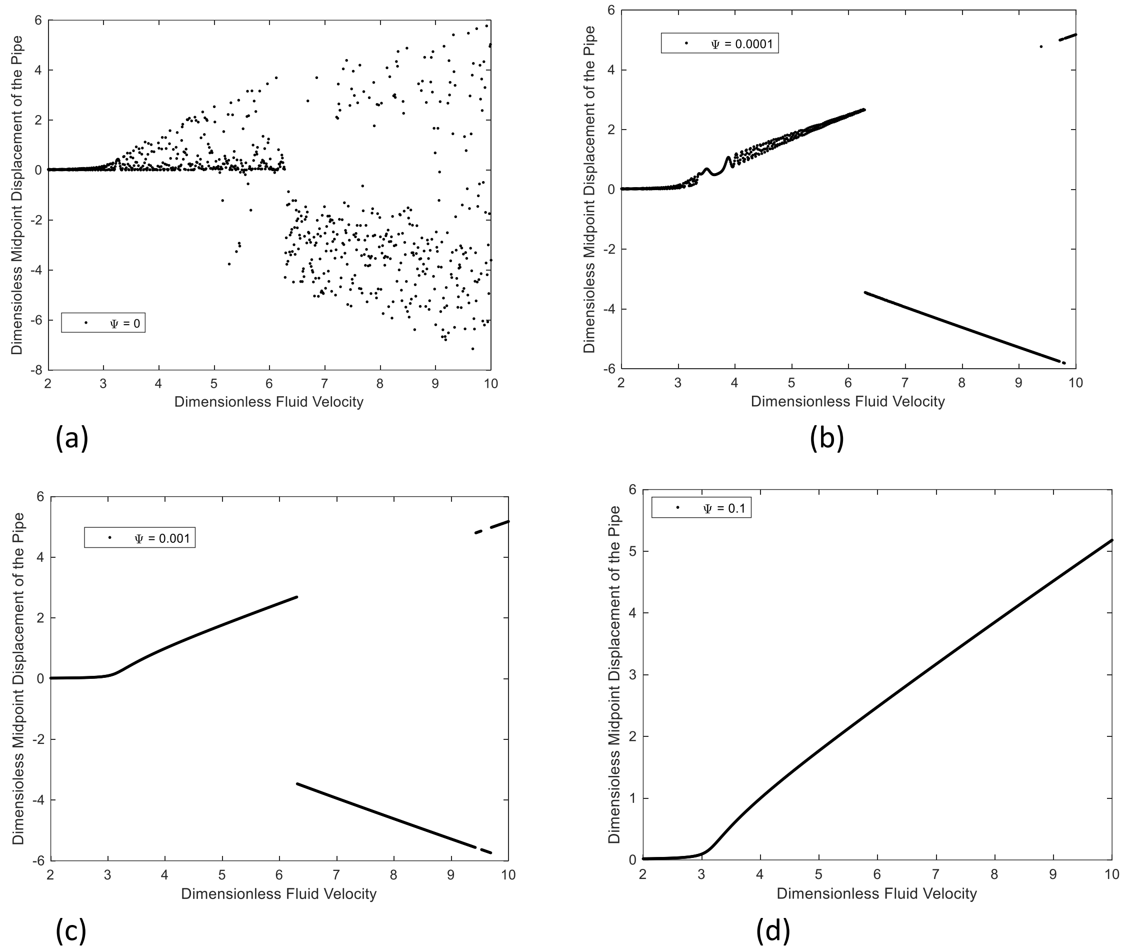


Figure 3. Plot of Midpoint Displacement of the Pipe against fluid velocity for viscoelastic damping (a) $\psi = 0$ **(b)** $\psi = 0.0001$ **(c)** $\psi = 0.001$ **(d)** $\psi = 0.1$

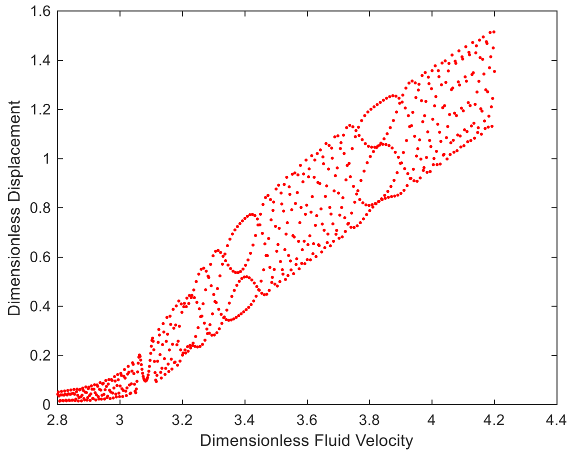
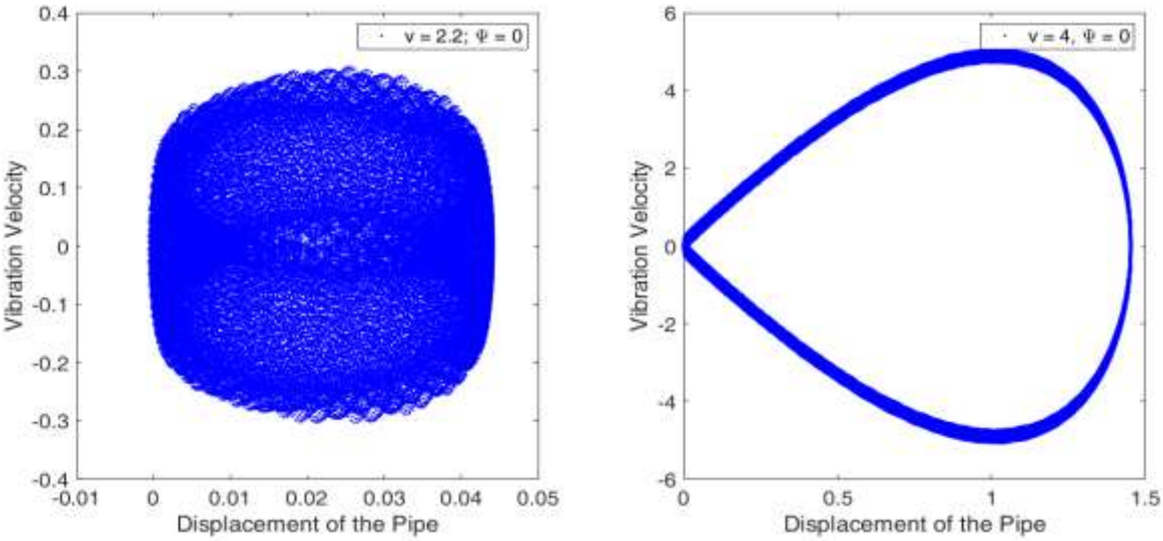
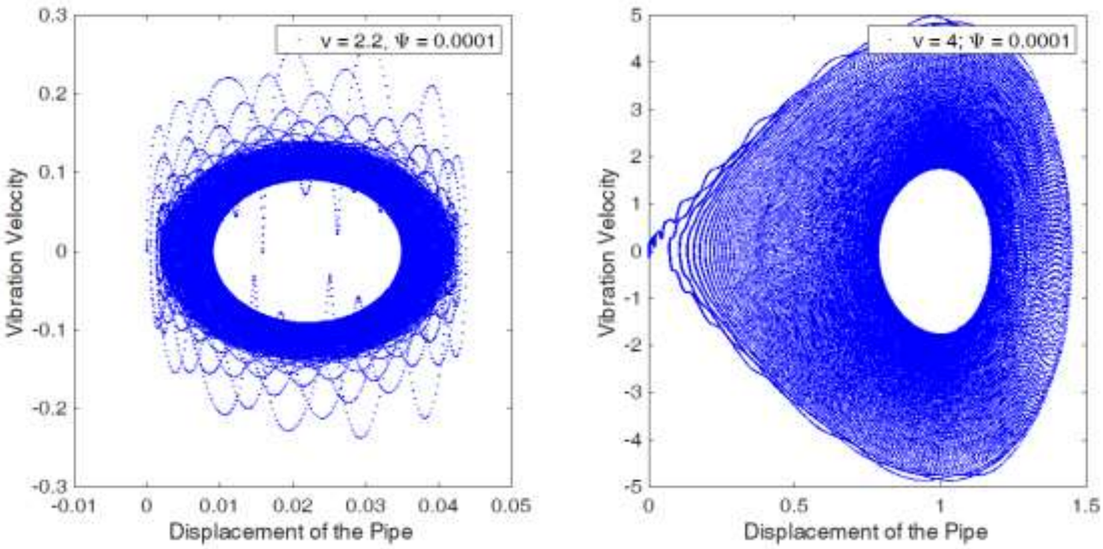


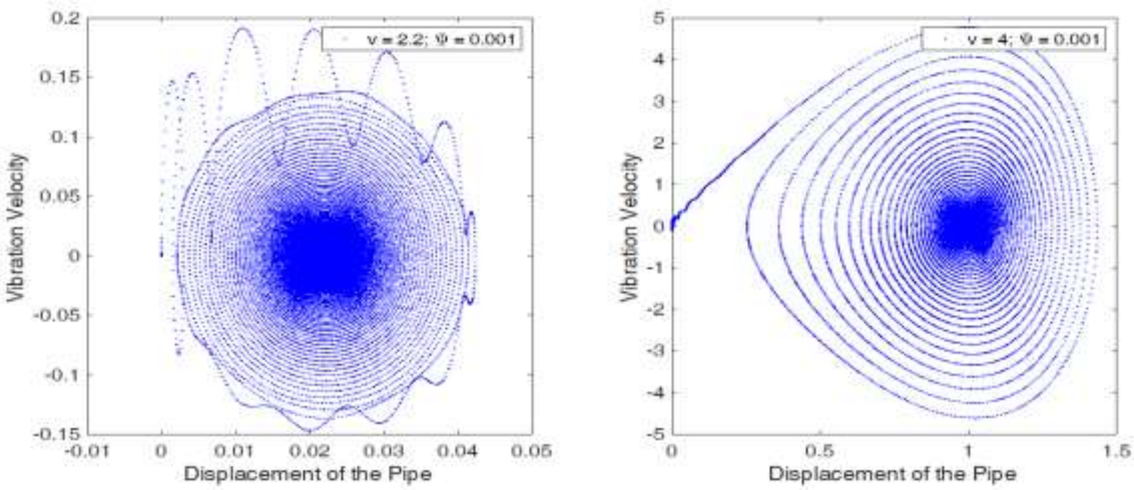
Figure 4. The Route to Chaos



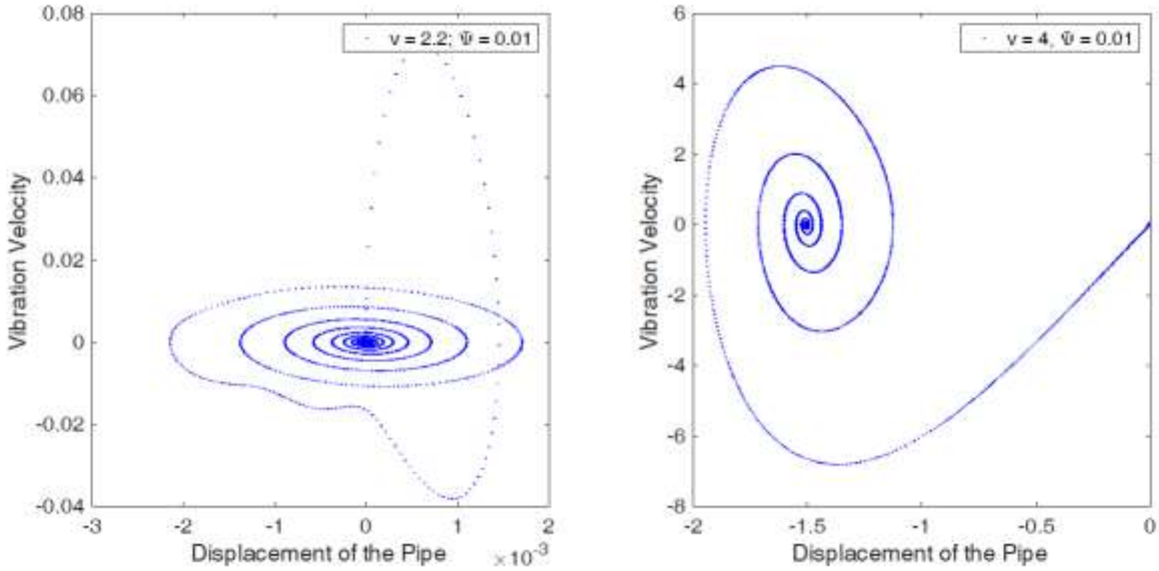
(a) $\psi = 0$



(b) $\psi = 0.0001$



(c) $\psi = 0.001$



(d) $\psi = 0.01$

Figure 5. Phase portrait of the pipe at v =2.2 (pre buckling region) and v=4 (post buckling region)

3.2. Effect of initial curvature on the Viscoelasticity on the Pipe Material

Viscoelastic pipes have Young's modulus much smaller than metal pipes. The pipe has an initial curvature altering its perfect symmetry. The imperfection is more pronounced when simply supported horizontally. Hence, considering Figure 6, the effect of initial curvature is shown. The buckling of a pipe without initial curvature is sharp as indicated by *b=0* in the figure producing a pitchfork bifurcation. However, when $b \neq 0$ there is a blurry of the sharp transition at the bifurcation point before it diverges rapidly with a lower displacement amplitude as *b* increases. Instead of a pitchfork bifurcation to take place as in the case of a perfectly straight pipe, cusp bifurcation results. This observation is in agreement with the experiment of Dodds and Runyan(1965) in which the pipe began to bow slightly as the flow velocity was increased as reported in Paidoussis (2014).

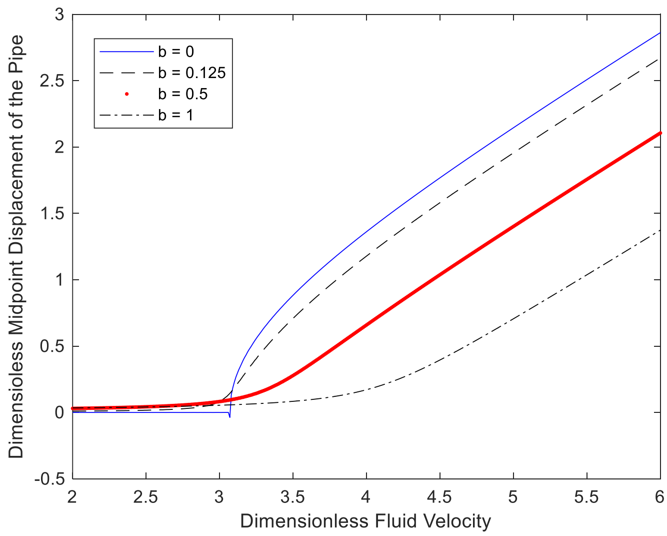


Figure 6. The plot of Midpoint Displacement of the Pipe against fluid velocity for various initial curvature

4. CONCLUSION

In this work, we have examined the stability of a slightly curved viscoelastic pipe conveying fluid. We have shown that the geometric imperfection has a significant effect on the viscoelastic pipe. The phase plane portraits reveal that viscoelastic damper attenuates the instability of the pipe, reduces the amplitude of vibration and increases the critical velocity. The bifurcation diagram shows that the route to chaos is through a periodic doubling.

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