Journal of Engineering Research, Special Edition, Volume 24, Number 1, December 2019

A Review on Mechanics of Impacts on Steel Plates

O. H. Aliyu*
Department of Building, Federal University Birnin Kebbi, PMB 1157, Kebbi State, Nigeria
Email: ohunene.hafsa@fubk.edu.ng

Abstract
This paper is a review of research works carried out within the last three centuries, describing the physical phenomenon that ensues when low and high velocity projectiles/missiles impact upon steel plates. It reviews the various developments in the experimental, analytical and computational models employed in evaluating local and global impact responses; work required for plastic deformation; minimum energy required for perforation as well as the determination of residual projectile velocities. It shows how an understanding of the material as well as the fracture mechanics of impact plays a crucial role in facilitating the accurate analytical and numerical predictions of impact phenomena.

Keywords: Damage mechanics, minimum and critical energy for perforation, residual velocity, ballistic limit and force time-histories

1.0 INTRODUCTION

Recoded incidences of accidental loading of structures through history for example: Rona point (1968), Flixborough (1974), Chernobyl (1986), Piper Alpha (1988), Peterborough (1989), Oklahoma City (1995) and Enschede (1998) have had a profound influence on design models and philosophy (Wen and Reid 1998; Watson and Alan 2002). The improvements in design against impact loads are as a consequence of realizing the various physical phenomena that takes place upon impact (Qiao *et al*., 2008). According to scholars such as Mughal *et al.* (1994), Christoforou *et al*. (2013) and Stronge (2018),'Impact effects can be divided into two categories, the overall response of the target structure and local effects' While Iqabal and Gupa's 2008; Wessman and Roses (1942, cited in Aliyu, 2019), considered the dominant effect to be the localized effects as the inertia of the structure as a whole does not give the structure sufficient time to react to the sudden high velocity impact loads. Jones and Paik (2012), have equally attributed the nature of the resulting impact effects to the magnitudes of the impacting velocities of the projectiles. They have stated that with low velocities, the global effect is dominant as there appears to be sufficient time for the target plate to contribute to the perforation process while for high velocity impacts, there appears to be an insufficient time for a build-up of global effects making the impact effects substantially localized. A number of these local effects tend to be specific to particular engineering materials. The local effects generated when a steel plate is subjected to impact loads may occur in several ways such as: petal formation (or dishing), ductile enlargement, plug formation and fragmentation (scabbing) of the target (Awerbuch and Bonder, 1973; BØrvik *et al*., 2001; Dey *et al*., 2004; Voyiadjis, 2013; Rosenberg and Dekel, 2016).

Scabbing: is the ejection of material from the back face of the target opposite the face of impact.
Spalling: is the ejection of material from the front face region surrounding the area of impact.
Penetration: is the depth to which a missile will penetrate into a target without passing through it.
Perforation: is the 'full penetration' or where the missile passes through the target with or without an exit velocity (Mughal *et al*., 1994).

According to Blackman and Goldsmith (1978); Rosenberg and Dekel (2016), the most common types of local failure effects for thin or intermediate steel plate targets subject to impact are as shown in figure 1.

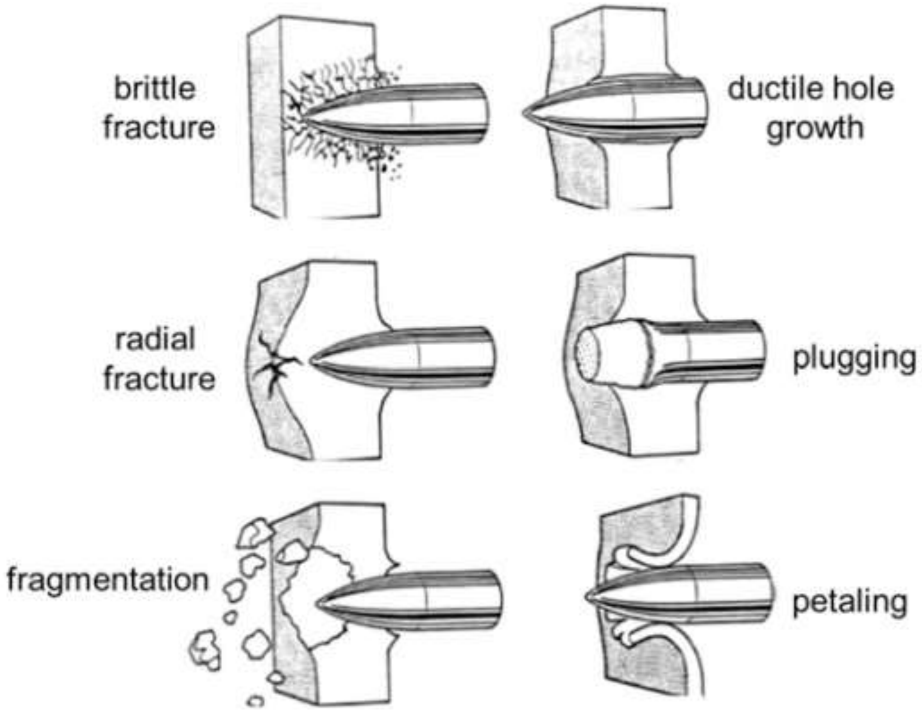


Figure 1: Perforation mechanism: adapted from Rosenberg and Dekel (2016)

Thus, the views of scholars such as Chen and Lang (2012) who from their works on *Perforation Mode of Metal Plates Struck by a Blunt Rigid Projectile* came to the conclusion that plunging is a likely mode of failure. The overall structural response on the other hand, encompasses the bending, shear and membrane responses as well as their induced failures, throughout the target (Borvik *et al*., 2001; Li *et al*., 2005), according to Corbett *et al*. (1996); Rosenberg and Dekel (2016), the methods available for gauging the effects on impact loaded steel structures are:

(i) Empirical methods
(ii) Analytical methods and
(iii) Numerical approach.

2.0 EMPIRICAL STUDIES

As mentioned earlier, the effects of projectile or missile impact on target can be classified as either local or global depending on the magnitude of the impacting velocities of the projectiles or missiles on the target steel plates which can be evaluated using a number of techniques as discussed above. The following sections, reviews empirical, analytical and numerical studies carried out in assessing both the local and global impact effects of target steel plates to projectiles/missiles.

2.1 Local Target Response

The evaluation of the local effects of projectile or missile impact is usually assessed by the use of empirical formulae which are derived from test programmes (Mughal *et al*., 1994). Earlier works on studying the phenomenon of penetration and perforation processes have been based on experimental investigations, with the experimental data obtained used in conjunction with analytical and dimensional considerations to describe the correlations that exists between the numerous factors involved (Corbett *et al*., 1996; Rosenberg and Dekel, 2016). As far back as the 18th century Robins (1742) demonstrated from his experimental investigations that 'if bullets of the same diameter and density impinge on the same solid substance with different velocities they will penetrate the substance to different depths, which will be in the duplicate ratio of those velocities nearly, and the resistance of solid substances to the penetration of bullets is uniform'. This breakthrough resulted in the renowned Robins-Euler method for calculating the estimated depth of penetration (Corbett *et al*., 1996). This discovery has been supported by the works of other scholars (Johnson, 1972; Blackman and Goldsmith, 1978), who have conducted similar researches into this particular area and have produced comparable experimental equations which are able to forecast such parameters as penetration depth and energy required to perforate a structure. Since experimental data are usually analysed using the dimensional analysis technique it can also be employed in obtaining empirical formulae (Sonin, 2001; Albrecht *et al*., 2013). Hopkinson (1915, cited in Christopherson, 1945) used this technique to replicate the effect of explosive loading of structures. With the use of this dimensional analysis technique, scaled models can be adopted in assessing the effect of impact which is cheaper than their prototype model counterpart (Corbett *et al*., 1996; Rosenberg and Dekel, 2016). Duffel *et al*. (1984) have used the scaling laws to assess the effects of low velocity impact loads on plates and came up with similar outcomes for both full and small scale models to within a margin of 10%, while Anderson *et al*. (1993) undertook a computational investigation to measure the consequences of scaling on the mechanics of penetration and perforation process observed in high velocity impacts and came to the conclusion that the resistance of small scale targets was somewhat higher when compared to that for a full size model which they attributed to the loading time as well as the size of the target plate rather than rate effects, although the variations were quite small (Rosenberg and Dekel, 2016). They were therefore of the opinion that more research should be carried out to accurately measure the effects of strain rate at the sub ordinance as well as the normal ordinance impact velocities which are usually within the ranges of (25$m/_S$ - 500$m/_S$) and (500$m/_S$ - 2000$m/_S$) respectively (Rosenberg and Dekel, 2016), before it can confidently be applied in the mechanics of penetration and perforation process. This however, confirms the views of scholars such as Jones (1984), who argued that the implementation of scaling laws on structures under dynamic loading particularly impact loads should not be used. He based his argument on the fact that strain rate effects and the existence of ductile brittle transitions are likely to occur either in the model or the prototype test version. Thus, the views of Dallard and Miles 1984; Booth *et al*. 1983; Rosenberg and Dekel 2016 who have also researched into the practicability of geometrical scaling to impact situations and came to the same conclusion. In Booth *et al*. (1993), Calladine tried to explain some of the variations that exists between the expected and actual behaviour of scaled models of steel plated structures loaded by dropped objects to be as a result of the scaling system adopted; concluding that that accuracy of predictions depends largely on the scaling variable range assumed, making them satisfactory for use in certain situations. More recently, Shadi *et al*. (2015) used the scaling laws based on the similitude theory to study the behaviour of plates subject to low velocity impacts and obtained results which were in agreement with experimental data. However, there is still a lack of sufficient experimental data to confirm the effectiveness of geometrically similar scaling laws for impact loading where strain rates and ductile brittle transition are most likely to occur thus limiting the accuracy of its application to static loading as well as low velocity impact loading. In recent times the assessment of steel plate structures to projectiles have been more inclined towards the use of analytical and numerical methods although, the use of empirical formulas in predicting the energy needed to go through target plates or barriers still plays an important role. Some of these formulas have been in existence as far back as the 18th century, though recent researches into this area of study have seen the emergence of new empirical formulae (Corran *et al.*, 1984; Corbett *et al.*, 1996; Jones *et al.*, 2008; Jones and Paik, 2012). The commonly known and the most frequently empirical formulae have been those of:

Robins–Euler formula for penetration depth (1742).

$$x = \frac{m_p {V_o}^2}{2a} \quad (1)$$

De Marre formula for the minimum energy required to perforate plate (1886).

$$E_c = a.{d_p}^{1.5}{h_o}^{1.4} \quad (SI) \quad (2)$$

The Stanford Research institute formula (SRI, 1963).

$$E_c = \frac{\sigma_u d_p}{10.29}\left(42.7{h_o}^2 + lh_o\right) \quad (SI) \quad (3)$$

Validity range:

$0.1 < {h_o}/{d_p} < 0.6; 0.002 < {h_o}/{l} < 0.05; 10 < {l}/{d_p} < 50; 5 < {l}/{d_p} < 8; {l}/{h_o} < 100; \quad 21 < V_o < 122\text{ms}^{-1}$

The Ballistic Research Laboratory (BRL) formula (1968) (Corbett *et al.*, 1996).

$$E_c = 1.44 \times 10^9 \left(h_o/h_p\right)^{1.5} \quad (SI) \quad (4)$$

The Bechtel Formula for Scabbing limit for steel pipe missiles (Sliter, 1980; Bangash 1993).

$$\frac{h_s}{d} = \frac{5.42M^{0.4}{V_o}^{0.65}}{{f_c}^{0.5}d^{1.2}} \quad (imperial) \quad (5a)$$

Or

$$\frac{h_s}{d} = 13.63\left(\frac{M^{0.4}{V_o}^{0.65}}{{f_c}^{0.5}d^{1.2}}\right) \quad (SI) \quad (5b)$$

The variable definitions for the equations are as shown in table 1.

Table 1: Variable definitions

Symbol	Parameter
x	Depth of penetration
m_p	Mass of projectile
A	Constant
V_o	Impact velocity
h_s	Scabbing limit
d_p	Diameter of projectile
M	Mass of projectile
σ_u	Ultimate direct stress
L	Target span
h_0	Initial target thickness
E_c	Minimum perforation energy
E_f	Energy required for first fracture

Nelson (1985) on the other hand, based on his investigations of the applicability of the SRI and BRL formula came up with a more manageable formula for long projectiles having used dimensional analysis in condensing the results. The formula he came up with is given by:

$$E_c = A\sigma_U {d^3}_p \left(\frac{h_o}{d_p}\right)^{1.7} \left(\frac{l}{d_p}\right)^{0.6} \quad (6)$$

For parameter ranges: 0.14 < $\frac{h_o}{d_p}$ < 0.64, 4 < $\frac{l}{d_p}$ < 22, and $\frac{L}{d_p}$ > 13.

Where, A is a constant which equals 1.4 for the calculation of the mean perforation energy and 1.0 for calculation of minimum perforation energy.

While Jowett (1986) brought together the data collected from a number of sources and came up with a bi-functional relationship which gives the minimum perforation energy for shorter projectiles named the Atomic Energy Authority Formula (AEA) short missile equations

$$E_c = 1.32\sigma_u {d^3}_p \left(\frac{h_0}{d_p}\right)^{1.74} \left(\frac{l}{d_p}\right)^{0.61} for\ 0.1 < \frac{h_o}{d_p} < 0.25 \quad (7a)$$

$$E_c = 0.38\sigma_u {d^3}_p \left(\frac{h_0}{d_p}\right)^{0.84} \left(\frac{l}{d_p}\right)^{0.61} for\ 0.25 < \frac{h_0}{d_p} < 0.64 \quad (7b)$$

Having validity ranges: 2 < $\frac{L}{d_p}$ $< 8.315 <$ σ_u < 483 MPa, 40 < V_0 < 200m/s, $\frac{l}{d_p}$ < 12. For $\frac{l}{d_p}$ > 12 the $\frac{l}{d_p}$ term should be replaced by unity (Corbett *et al.*, 1996). Wen and Jones (1992), showed the importance of using empirical formulae for impact conditions that are within their stated range of application. They were able to demonstrate from the work they carried out on low velocity impact of mild steel plates using blunt and flat faced penetrators that the Nelson, SRI and AEA short missile equations gave values which were greater than the critical impact energy required for perforation of plates as the test parameters used in the investigation were outside the range of applicability of these equations. The BRL formula on the other hand, gave a more precise estimate of critical impact energy. Wen and Jones (1992) also came up with a new formula for finding the critical penetration energy which is based on the principles of dividing the energy absorbing mechanism of the plate into two components: a local and global component.

$$\frac{E_c}{\sigma_u {d^3}_p} = \left(\frac{2\sigma_y}{\sigma_u}\right)\left[\left(\frac{\pi}{4}\right)\left(\frac{h_o}{d_p}\right)^2 + \left(\frac{l}{d_p}\right)^{0.21}\left(\frac{h_0}{d_p}\right)^{1.47}\right] \quad (8)$$

This gave good valid predictions of the critical perforation energy for mild steel plates impacted upon by blunt flat faced indenters. So that for high mass low velocity impacts where the overall global plate response is important in evaluating plate response the above method can be employed particularly in cases where adiabatic shearing does not occur giving as an example, the impact of plates by dropped objects. Corbett and Reid (1993) worked on quasi-static and dynamic loading of monolithic simply supported steel plates and observed the importance of local indentation to the plate's response when penetrated by hemispherical–ended and flat–faced indenters. The results of their test were compared with results from empirical predictions obtained using the SRI and BRL equations, the Nelson equation and the AEA short missile equation. The SRI and the Nelson equations however, gave reasonably good results for the least energy necessary to perforate the steel plates, given the fact that it was derived from tests using flat-faced projectiles and outside the stated range of acceptability for these formulae. Very few empirical formulae have been derived for the critical impact energy for pipes and tubes, Stronge (1985 sited in S.R Reid 2016) however, fitted a power law to experimental data obtained from impact test on steel tubes and produced a relationship for deriving the critical impact energy

$$E_f = 1.7{h_0}^{2.0}{d_p}^{0.8} \quad (9)$$

$$E_c = 1.1{h_o}^{1.63}{d_p}^{1.48} \quad (10)$$

Wen and Reid (1998) put forward an approximate theory to predict the deformation and perforation of metallic cylindrical shell struck normally by blunt projectiles. The results obtained using the proposed theory compared closely with the experimental results on the cylindrical shell struck transversely by the same projectile when the material strain rate sensitivity was considered. This gave the critical impact energy for perforation as well as predicting the maximum displacement.

In considering aluminium plates, Jones *et al*., (2008) came up with the dimensionless perforation energy for aluminium alloy plates as:

$$\Omega_p = {}^{\pi\psi}/_{4} + 0.1(S/d)^{0.6}\psi^{1.3} \quad (11)$$

With a validity range of $1.4 < \psi < 4$ while Corran *et al*. (1984), proposed a non-dimensional equation for calculating the penetration energies of aluminum alloy plates within an impact velocity range of $43 < V_O < 145 m/s$ given as:

$$\Omega_p = \frac{G(34.79H)^2}{2\sigma_Y H^3} \quad (12)$$

With a validity range of $1.8 < \psi < 6$

Where:

$\Omega_p =$ Perforation energy; $\psi =$ ratio of projectile diameter to plate diameter; S = span of plate and d = diameter of projectile.

More recently, Jones and Paik (2012) carried out experimental investigation on the perforation of aluminium alloy plates with a range of projectiles with numerous faces that were ideally flat with low and moderate velocities. They have stated that the addition of an extra trial data to equation 11 results in a new expression for the perforation energy given as:

$$\Omega_p = \{1 + 0.2(\psi - 2)\}\,\{\pi\psi/4 + 0.1\,(S/d)^{0.6}\psi^{1.3}\} \quad (13)$$

which increases its validity range to $2 < \psi < 10$.

They further went on to state that since aluminium is less strain sensitive compared to mild steel, using the BRL equation directly overestimates its perforation energy (Ω_p). However, multiplying the BRL equation by 1/4 gives a resulting expression for calculating the dimensionless perforation energy for aluminium plates as given:

$$\Omega_p = \left(0.35\, x\ ^{10^9}/_{\sigma_y}\right)\ (\psi)^{1.5} \qquad (14)$$

While dividing the BRL equation by 2 gives:

$$\Omega_p = \left(0.7\, x\ ^{10^9}/_{\sigma_y}\right)\ (\psi)^{1.5} \qquad (15)$$

The results from equation (14) gave quite small values while the results from equation (15) gave good approximation of the perforation energy for moderate impact velocities within the validity range of $\psi < 15$. Above this range (i.e. $\psi > 15$), the BRL equation will have to be multiplied by 3/4 to give good approximation with experimental data for moderate impact velocities.

3.0 ANALYTICAL STUDIES

Empirical formulae though very important to this area of study, suffer some major draw backs. Parameters such as penetration depth, scabbing and perforation limits are often formulated using curve fitting test data, most of which are unit dependent. In most of these cases however, the range of validity is limited to the extent at which the test data was acquired which tends to create difficulties. For example:

The dimensional inconsistency of the formulae makes the comparism between different experimental results and between experimental and analytical predictions difficult.
The limitations on the range of validity of empirical formulae cannot be completely justified.
Finally, the current definitions of projectile nose shape factors (see figure 3) as has been used in many empirical formulae is open to a number of interpretations which introduces uncertainties in the evaluation of local impact effects.

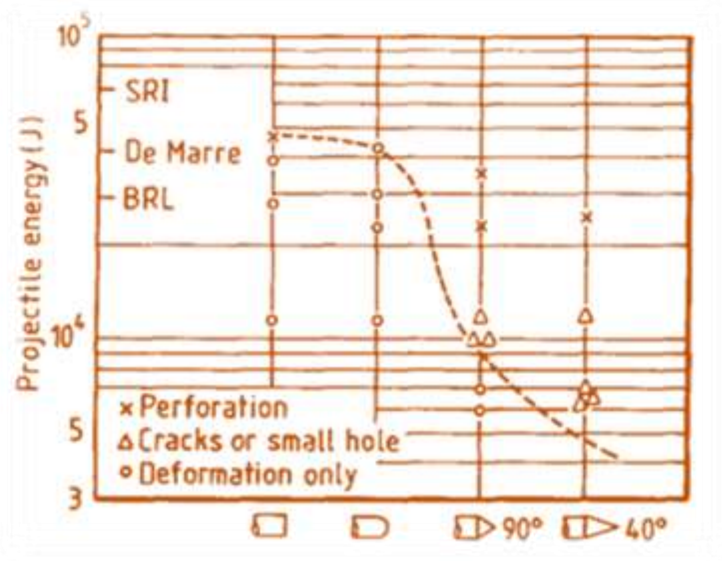
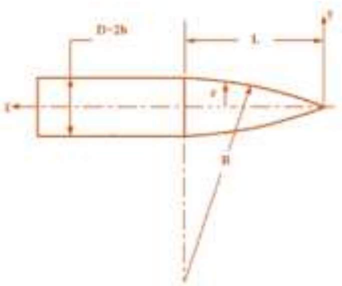


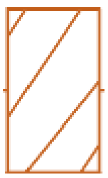
Figure 3: Empirical design graph showing impact test results on 7mm thick steel target plates using flat-faced, hemispherical-ended and conically-nosed cylindrical projectiles, adapted from Ohte *et al.* (1982).

The analytical models however, tend to be simpler and more accurate. For such models, the idea behind the mechanics of local effects of missile impact appears to be better understood. The complex relationships that exist between material and configurational parameters as well as the material flow history are better modelled. Adopting this approach gives a more efficient and economic way of assessing the local effects of missile impact and in addition extending the range of applicability of empirical formulae. The ambiguity surrounding the definition of projectile nose shape factor as evident in most empirical formulae has been removed following the introduction of the simplified well-defined analytical model (Li *et al.*, 2005).

Spherical truncated



Ogive



Flat

Figure 4: Shows nose shaped possibilities for spherical, truncated oval (blunt), ogive (sharp) nose and flat nosed projectiles adapted from (Li *et al.*, 2005 and Cheng *et al.*, 2007).

Goodie (1965) developed an analytical model for predicting the local effects of penetration depth by using the dynamic spherical cavity-expansion principles. In his prediction model, target inertia was included enabling him to approximate the target response. Warren and Poormon (2001) equally, developed an experimentally validated analytical forcing function from the dynamic spherical cavity-expansion principle for evaluating the target response to ballistic events. This involved multiplying the spherical cavity forcing function by a decay function to better describe the occurrence of free surface effects during oblique penetration - see equation 11.

$$\frac{\sigma_r(a)}{Y} = \left[A + B\left(\sqrt{\frac{P_O}{Y}}\,\dot{a}\right) + C\left(\sqrt{\frac{P_O}{Y}}\,\dot{a}\right)^2\right] f(d, a.) \qquad (16)$$

The above experimentally validated analytical function was used in combination with the explicit transient dynamic finite element code to model the projectile as well as the target response respectively. The results obtained from the simulation agreed excellently with the experimental results.

Chen and Li (2003) on the other hand introduced a closed form analytical solution. In their model, the local indentation/perforation was evaluated using the dynamic cavity expansion equations while the effect of structural response was considered via rigid plastic analysis. The validity of this model was verified in the works carried out by Borvik *et al*. (2003) where the same jump in residual velocity was observed at ballistic limit. Ben-Dor *et al*. (2010) conversely, adopted a broad-range of semi empirical (approximate) models in assessing the result of layering on the ballistic properties of metallic targets against shape nosed projectile impact. The result obtained agreed with both experimental and numerical results. Other methods adopted by scholars in analytically analysing local effects of missile impact are those of the:

Energy balance method and
Conservation of momentum method (Awerbuch and Bonder 1973).

The earliest effort towards analytically investigating the mechanics of penetration using the energy and momentum method has been accredited to the work carried out by Bethe on the static analysis of the penetration process. Taylor (1948) however, further improved on this based on the enlargement of a circular hole by a conical head projectile perforating a thin plate, he was able to derive an expression for the total work required for plastic deformation (see equation 17).

$E_c = 1.33\pi {r_p}^2 h_o \sigma_y$ (Awerbuch and Bonders, 1973). (17)

While Beth had tried to relate the stress in the plate to its deformation, Taylor understood that with the disparities in ratios of the principal stress all through the penetration process, the only justifiable relation will be to relate the stress to the strain increments. This association was used alongside the Von Mises yield criterion to set up the stress distribution in the plate and the work necessary for perforation. This however, resulted in a value much lower than Beth's value (see equation 18).

$$E_c = 2\pi {r^2}_p h_o \sigma_r \quad (18)$$

Where:

$E_c =$ Minimum energy required for perforation, $\sigma_r =$radial stress and

$h_o =$ Inital target thickness

Although, Taylor did not consider the effects of inertia in his analysis, scholars such as Frieberger (1952) and Kumari (1975), further extended the theory to include this effect. Thompson 1955, cited in Awerbuch and Bonder 1973 in adopting this energy balance method, used a different approach where he assumed that 'the reduction in projectile kinetic energy is as a result of the work done by the projectile in plastic deformation' (BRL Report 1977). He however, arrived at a similar expression which gave the energy dissipated due to plastic deformation, heating and inertia resistance of the target material. In considering the dynamic effects, Thomson investigated the penetration and perforation of plates by projectiles of multiple profiles and came up with an expression for estimating the residual projectile velocity, V_r given by equation 19 as:

$$V^2{}_r = V^2{}_o - \frac{4\pi r^2{}_p h_o}{m_p}\left[\frac{\sigma_y}{2} + \frac{\rho V^2{}_0}{3}\right] \quad (19)$$

Scholars such as Srivathsa and Ramakrishnan (1997) also used the energy balance method, and arrived at a ballistic performance index for estimating and comparing the ballistic qualities of metal materials. In adopting the concepts of momentum balance, Zaid and Paul (1957 and 1958) extended the method used by Thomson to determine the residual velocity of perforating projectiles after the normal impact of a thin plate to give the relationship in equation 20.

$$V_r = \frac{m_p V_o}{m_p + 2\pi\rho h_o r^2{}_{p\,sin\beta}} \quad (20)$$

Where:

$m_p =$ Mass of projectile, $V_0 =$ Initial velocity, $V_r =$ Residual velocity,

$\rho =$ Mass density, and $r_p =$ Projectile radius

Which they applied to various truncated conical and ogive nose shapes and compared the results they obtained with experimental data. Their method however required that the final shape of the perforated plate be specified. For most of the proposed analysis techniques, assumptions were based on:

Constant velocity during the perforation process of thin plates.
The absence of plastic deformation beyond the immediate zone surrounding the whole.
A constant pressure on the projectile.

For these reasons research has been limited to the case of high velocity impact (i.e. appreciably higher than the ballistic limit). Recht and Ipson (1963) used the concept of conservation of momentum and energy balance to investigate the mechanics of penetration by projectiles. Based on their energy analysis method they were able to predict the residual velocity provided that the minimum perforation velocity was known (see equation 21 and 22).

$$V_r = \frac{m_p}{m_{p+m_{p1}}}\left[V_o{}^2 + V_x{}^2\right]^{\frac{1}{2}} \quad (21)$$

This can further be reduced to:

$$V_r = \left[V_o{}^2 + V_x{}^2\right]^{\frac{1}{2}} \text{ if } m_p \gg m_{p1} \quad (22)$$

Dey *et al.* (2007), adopted this conservation laws in investigating the ballistic resistance of double layered steel plates at sub-ordinance velocity. In their study, the ballistic limit velocity was already determined from experimental tests. The outcome of their investigation was that the minimum ballistic limit velocity obtained increased substantially by double layering of target plates.

According to Corbett *et al.* (1996), the energy momentum balance method is only useful when calculating the velocity drop in projectiles striking a target at well above the ballistic limit and that when, the impact velocities is near to or at the ballistic limit it generates responses in the target material that substantially influence the penetration process and in such a situation the energy and momentum balance method will not be suitable for use. Most of the studies discussed so far have been limited to the case of non-deformable projectiles (hard missiles) with a focus on only one possible method of perforation. However, the actual perforation of target plate may occur in a sequence of two or more processes. This makes the energy and momentum balance method quite limited in application as it does not give an accurate insight into the actual perforation process (Corbett *et al*., 1996). Researchers such as Awerbuch (1970) and Goldsmith & Finnegan (1971) were of the view that the perforation process should consider a number of deformation mechanics acting at different stages. In their view, a consideration of various types of deformation mechanics on the whole would be more representative of the actual circumstance. This was the basis of the preliminary investigations carried out by Awerbuch (1970) and later expanded by Goldsmith and Finnegan (1971). Awerbuch and Bonders (1974) sought to further develop these investigations and resolve a number of limitations, in so doing they presented an analysis technique which fairly but accurately predicts the post perforation velocities, contact times and force–time histories. This analysis nonetheless, relies on a small number of experimental quantities which can be established from a few trial tests. Once established for a particular projectile and target material the predictions can then be achieved for a wide range of projectile velocities and target thickness.

Based on these laws of conservation, Liang *et al*., (2005) came up with their own analytical model of investigating the ballistic perforation resistance of targets. The results obtained from their model gave good agreement with the experimental data obtained from Almohandes *et al*. (1996).

3.1 Semi Empirical Studies

According to Mughal *et al.* (1994) the response of a structure subject to dynamic load depends not only on its dynamic properties but also on the nature of the applied loading. Therefore, in assessing the response of structures subject to impact loading, one of the critical factors to consider is the measurement of the impact force. Impact loads are considered to be dynamic in nature and can be defined in the form of a force-history usually referred to as the force-time history of the applied loading and are usually characterized by a rise time of less than a second making it quite hard to measure and even harder to predict (Corbett et al., 1996; Johnson 1972, cited in Aliyu 2019). One of the earliest attempts towards assessing this force can be seen in works done by Robins in 1742, 'The New Principles of Gunnery'. Robins suggested that the resisting force of a target subject to impact load is constant throughout and is independent of speed and depth of penetration. Poncelet (1835) however, further modified on this by suggesting that there is a need to take into consideration will be necessary to overcome cohesion of the target material. To this effect he suggested that the original expression by Robins in predicting the impact force should include a term which is proportional to the square of the velocity. A third term was further included proportional to the projectile velocity to represent the frictional resistance. This resulted in a semi empirical formula for measuring the impact force given in equation 23 (Corbett *et al*., 1996).

$$F = a_1 + a_2 V_o + a_3 V^2{}_o \quad (23)$$

Experimentally obtained force-time histories have allowed empirical formulae to be fitted to the experimental data (Corbett *et al*., 1996). This view was shared by Kar (1978b) who argued that though different methods exist for the determination of dynamic load, the use of load-time history is usually more appropriate as the results obtained from this method compares excellently with full scale test results. He also suggests that while using this method appropriate load time history graph should be chosen which will be appropriate for the problem under consideration, as there is no one to one relationship between load function and depth of penetration. In his work he used a triangular deceleration function with equal rise and decay times to obtain the load time history, he emphasizes that this may not be appropriate in other cases. Levy and Goldsmith (1984a, 1984b) derived an expression for force-time history for normal impact of thin plates by hemispherical ended projectiles. Their expression gave a good estimate for results below ballistic limit but required measured information for situations above the ballistic limit but because the terms in the expression were treated in lumped parameters, it did not particularly yield any stress or strain. It however, gave a simple and effective method for predicting the force generated during impact and corresponding target displacement. Finding the force-time history however, led to solving the equation of motion as given in equation 24.

$$-m_p \frac{d^2 w_c}{{d_t}^2} = m_{eq} \frac{d^2 w_c}{dt^2} + B \frac{d_{w_c}}{d_t} \qquad (24)$$

Where: m_{eq} =equivalent mass and w_c = deflection

In reviewing the successful experimental methods used in obtaining the force time history for dynamic loads, Virostek *et al.* (1987) also proposed a force-time history for a hemispherical–tipped projectile for any angle of incidence given by equation 25:

$$F_{(t)} = A_{(t)} \left[\sigma_y + \frac{1}{2} C_D \rho V^2(t) \right] \qquad (25)$$

Where: $A_{(t)}$ is the projected area on the target in the direction of travel at the time t and C_D is the drag coefficient.

3.1.1 Global target response

As mentioned earlier structural responses to impact loads not only include local effects but also include the overall structural response. According to Mughal *et al.* (1994) the overall global response of the target structure can be assessed using any one of the following methods.

(i) Force-time history solution which involves the numerical integration of the equation of motion. This is the general method applicable for any pulse shape (force/function) and resistance function.
(ii) The response chart solution, which can be employed provided that the idealized pulse shape (interface forcing function) and the resistance function are compatible with the response chat.
(iii) The energy momentum balance solution which is used when the interface forcing function cannot be defined or where an upper bound check on structural response is desired. This method is best suited for impact loading and depends on the missile type and target.

According to Jones (1993), when impact loading of a structure or its elements are caused by dropped objects, then the impact velocity can be determined using the energy balance method. He also suggests that since inertia forces are small, displacement response of the target structure due to mass impact loading can be estimated using the quasi-static method which applies the principles of energy balance in equating the work done by the static concentrated load at the impact location to the displacement suffered by the target. In examining the adequacy of this method, he investigated the case of a fully clamped beam struck by a mass at the mid span and came to the conclusion that this approach over predicts the maximum permanent transverse displacement. He attributed this to the energy absorbing mechanism which is significant when the striker mass is smaller than the total mass of the beam, implying that for this method to be employed the mass ratio of the striker to beam must be greater than one. Further studies carried out by Wen and Jones (1993b) suggests that the quasi-static method is only valid when the impact velocity is less than a value lying within the range of 10-20m/s. Jones (1993) also proposed that for a fully clamped beam impacted upon by a dropped object the overall structural response can be determined using the rigid plastic method provided that account is taken of the materials strain rate sensitivity. The application of this method gave results that compared well with the experimental test results on mild steel beams which are highly strain-rate sensitive. In investigating the case where the fully clamped mild steel beams were struck at different locations along its span by relatively heavy masses, Jones and Jones (2002) used this plastic method approach and the results they obtained correlated well with experimental results. They were able to show that a substantial reduction in threshold energies occurred when the striker mass struck close to the beam support. The round nosed impactors however, required more energy to cause failure than the flat nosed ones with the support failures being more pronounced as opposed to rupture at impact position. More recently, Aliyu (2019) investigated the integrity of steel beam under impact to prevent brittle fracture. In her study, she adopted the energy balance method suggested by Jones (1993) in evaluating impact velocities due to dropped weights (which were within the ranges of 17.16-22.15m/s). This was used alongside the energy momentum balance method suggested by Mughal et al. (1994) for evaluating the upper bound estimate of structural response which took into account the effect other loadings on the structure has on the ductility capacity. The study required verifying the displacement at which the available strain energy of the system equalled the kinetic energy after impact. She was able to obtain from her analytical calculations, the maximum allowable deflection for the steel beam which was verified numerically using the transient dynamic finite element code.

3.1.2 Interface forcing function

According to Mughal *et al.* (1994) the pulse shape of any force time-history function depends on the type of missile and on the nature of impact which can be classified either as

Elastic impact or inelastic impact.

In an elastic impact it is assumed that none of the kinetic energy is lost so that the missile and structure deform elastically remaining in contact for a short period of time and then rebounds due to the action of elastic interface restoring force. While, for a plastic impact, the missile, structure or both may deform plastically (completely inelastic) sustaining permanent damage or deformation with the bodies sticking together on impact and remaining in permanent contact (Muncaster, 1993; Stronge, 2018). Similarly, according to Newton's experimental law of impact, classification can also be based on the coefficient of restitution 'e', with an elastic impact having a unit value of 1 and a value of less than one for a completely inelastic impact (Stronge, 2018). This is shown in the table 2.

Table 2: Classification of collision adapted from Muncaster (1993)

Coefficient of restitution$n(e)$	Deformation characteristics
1	Elastic
< 1	Elastic plastic
0	Plastic (completely inelastic)

4. NUMERICAL STUDIES

With computer modelling and simulations now being regarded as a rising 'third pillar' of 21st century science and engineering, it has become commonplace to numerically simulate high velocity impacts using powerful computer codes called hydrocodes which are well equipped to handle excessive temperatures and pressures, rise times of very short duration as well as large displacements using discretization method, finite difference or finite element codes (Zukas and Scheffler 2000; Kane *et al*., 2009; Voyiadjis, 2013; Mohotti *et al*., 2013 Rosenberg and Dekel 2016). These numerical simulations of structures subject to fast transient load are considered crucial, as they are able to provide information (on the stress and deformation fields) which are far more detailed than those obtained from any experiment as they tend to investigate not just the process involved but the physics behind the process as well. The information so obtained, are frequently employed to improve the resistance of the structures to these loads. A number of powerful numerical hydrocodes are however, commercially available for impact simulations and these includes the LS–DYNA, AUTODYN, and ABACUS (Wen 2000; Voyiadjis 2013; Mohotti et al., 2013). These simulations run algorithms based on discrete stages of interactions between material element and particle (Li *et al*., 2005). For all materials involved in the simulation process, two sets of data must be given to help describe the behaviour of the continuum i.e. the equation of state for the material as well as the constitutive relations solved using either the finite difference or finite element code (Rosenberg and Dekel, 2016). Initially, the method had some limitations in the sense that the simulation of failure process was concentrated on a single failure mode which is not actually the case (Anderson and Bodner, 1988). These limitations according to De Rouvray *et al.* (1984) and Voyiadjis *et al.* (2009) were attributed to the need to model not just the fracture mechanisms but also the material failure which requires a method for representing both the failure and its propagation which according to Arias *et al.* (2008) and Rusinek *et al.* (2008) is a relatively complex process. The initial codes available did not contain equations complex enough to describe the materials behaviour involving low velocity impacts and the criteria governing the failure processes was extremely difficult to model accurately particularly when the dynamic properties of the missile and barrier are unknown Jonas and Zukas (1978). The current advancement in the above simulating technologies (i.e. LS–DYNA, AUTODYN, ABACUS was well as the use of multiscale method also known as computational homogenizational technique) has made it possible to overcome some of these limitations. With these advancements, BØrvik *et al.* (1999) performed numerical investigations on the ballistic perforation of Weldox 460E, steel plate of 12mm thickness in order to examine the resulting impact phenomena. For their numerical simulations, they made use of the Johnson-Cook constitutive equation and fracture criterion where they were able to show that beyond the ballistic limit velocity, the residual velocity increased in a non-linear fashion to the increase in impacting velocity. In a similar fashion, Kane *et al.* (2009) adopted the modified form of the Johnson-Cook constitutive equation and fracture criterion (which made allowance for large plastic strains, high strains and adiabatic heating) in investigating the decrease in ballistic limit velocity with increasing target strength for Weldox 460E, Weldox 700E and Weldox 900E steel struck by blunt nose cylindrical steel projectiles. Although, slight variations between the experimental and numerical predictions were observed, the agreement between both sets of results was still with the experimental range and hence, considered acceptable. Dey *et al.* (2004) on the other hand, used the non-linear LS-DYNA finite element code in studying the consequences of target strength on perforation of Weldox 460E, Weldox 700E, Weldox 900E, steel plates struck by ogive, conical and blunt shaped projectiles; and they also made use of the Johnson-Cook constitutive equation and fracture criterion. The code give excellent description of the physical mechanism in the perforation process and they were able to arrive at the conclusion that the various projectile nose shapes as well as the material strength greatly influenced the ballistic limit velocity of the target plates.

The studies so far have been based on the impact of large size projectiles on thin as well as thick target plates, with very little numerical information in literature describing the impact effects of small size projectiles on metallic plates. To this end, Pradhan *et al.* (2017) recently carried out numerical simulations using AUTODYN to investigate the impact effects of small size spherical projectiles of 10mm diameter on a 4mm thick mild steel plate where they have also adopted the Johnson-Cook model and failure criterion. From their simulation results the response on the target plate to the projectile impacting velocity was explained in two phases. On the one hand, the residual velocity was seen to diminish well away from the minimum value (which was obtained from the work of Goldsmith and Finnegan 1971) while in the second phase, the residual value was seen to rise above the minimum value when the impacting velocity of the projectile was increased. These results were found to be in good agreement with the experimental data obtained for the works of Goldsmith and Finnegan (1971).

With computational homogenizational technique however, accuracy and efficiency is taking advantage of by coupling both microscopic and macroscopic constitutive models where the microscopic constitutive behaviour is defined as computations are taking place (Feyel and Chaboche 2000; Kouznetsova *et al*., 2001). This makes it possible to accurately model the material behaviour at high stain rates and temperatures. Voyiadjis *et al*., (2009); Karamnejad and Sluys (2014) employed this multiscale simulation technology in simulating deformations and failure in the critical regions of high stress and strain gradients in composite materials under high frequency loading with very good results. While Voyiadjis *et al*., (2009), in their study developed a micromechanical constitutive model that coupled the anisotropic damage mechanics with viscoplastic damage model of the material matrix which in their opinion should accurately simulate penetration and perforation of laminated composite metal to impact loads.

5. CONCLUSION

The following conclusions can be drawn from this study:

1. Experimental data used in conjunction with dimensional analysis technique to arrive at empirical formulae, are quite capable of predicting parameters such as penetration depth and energies required for perforation of structures, provided they have been used within their stated ranges of application.
2. The use of scaled models in assessing the effect of impact while cheaper than their prototype counterparts are not quite accurate. As the effect of strain rates have not been properly accounted for.
3. The use of analytical formulae on the other hand appears to be simpler and accurate as the mechanics behind penetration and perforations are better understood and modelled.

4. With analytical methods the ambiguities surrounding the definition of projectile nose shapes has been eliminated.
5. For numerical simulations, it has been shown that it is the ability of hydrocodes to handle excessive temperatures and pressures, rise times of very short duration as well as large displacements using discretization method, finite difference or finite element code that is key.
6. Similarly, the description of the continuum behaviour by accurately modelling both material and fracture damage at high strain rates and temperatures is crucial as it gives a more precise representation of the actual effects of the impact event.
7. The accurate formulation of these models enables us generate hypothesis and make predictions regarding fast transient loading of structures at a level of details and time scale which were initially not considered feasible experimentally.

REFERENCES

Aliyu, O. (2019). Design Recommendations for Steel Beams to Prevent Brittle Fracture. *Nigerian Journal of Technological Development*, 16 (2): 78-84.

Alves, M. and Jones, N. (2002) Impact Failure of Beams Using Damage Mechanics Part II Application. 27 (8): 863-890.

Albrecht, M.C., Nuchtshem, C.J., Albrecht, T.A. and Cook, R.D. (2013). Experimental Dimensional Analysis. *Technometric*, 55 (50).

Anderson Jr, C.E., Mullin, S.A. and Kuhlman, C.J. (1993). Computer simulation of strain rate effects in replica scale model penetration experiments. *Int. J. Impact Engng* 13: 35-52.

Anderson and Bodner (1988). Balistic Impact: The Status of Analytical and Numerical Modelling. *International Jorunal of Impact Engineering.*

Arias, A., Rodriguez-Martinez, JR., Rusknek, A. (2008). Numerical simulations of impact behaviour of thin steel plates subject to cylindrical conical and hemispherical non- deformable projectiles. *Eng. Fract. Mech.,* 6 (75):1635-56.

Awerbuch, J. and Bunder, S.R. (1974). Analysis of the mechanics of perforation of thing plates by hemispherically tipped projectile. *Int J Solid Structures* 10: 671-684

Awerbuch, J. (1970). A mechanics approach to projectile penetration *Israel J. Tech* 8:375- 383.

Baker, JF. (1943). The design of war time structures. *The Civil Engineer at war, Inst of Civil Engrs*, Vol 3.

Bangash, M.Y.H. (1993). *Impact and Explosion: Structural Analysis and Design*. Boca Raton, FL: CRC Press

Backman, M.E. and Goldsmith, W. (1978). The mechanics of penetration of projectiles into targes. *Int. J. Engrg. Sci.* 16: 1-99.

Bangash, M.Y.H. (2009*) Shock Impact and Explosions Structural Analysis and design* London UK.

Ben-Dor, G., Dubinsky, A., Elperin, T. (2006). Effects of air gaps on the ballistic ressitance of ductile shields perorated by non-conical impators. *Journal of Mechanics of Materials and Structures,* 1 (2): 279-299.

Ben-Dor, G., Dubinsky, A., Elperin, T. (2010). Effect of layering on ballistic properties of metallic shields against sharp-nosed rigid projectiles. *Eng. Fract. Mech.,* 77: 2791-9.

Booth, E., Collier, D. and Miles, J.C. (1983) In structural crash worthiness (Jones, N and Wierzbicki,T. (ed)) *Butterworth*: 136-174.

Borvik, T., Langseth, M., Hopperstad, O.S., Malo, K.A. (1999). Ballistic Penetration of Steel Plates. *Int. J. Impact Engng,* 22: 855-886.

Borvik, T., Hopperstad, O.S., Langseht, M., Malo, K.A. (2003). Effect of Target Thickness in Blunt Projectile Penetration of Weldox 460E Steel Plates. *Int. J. Impact Eng. ,* 4(28): 413-464.

Borvik, T., Langseth, M., Hopperstad, O.S., Malo, K.A. (2001). Perforation of 12mm thick steel plates by 20mm diameter projectiles with flat, hemispherical and conical noses part I: Experimental study. *International Journal of Impact Engineering,* 27: 19-35.

Chen, X., Li, Q.M. (2003). Plugging and perforation of ductile circular plates struct by a blunt projectile. *Int. J. Impact Eng. ,* 5 (28): 513-536.

Christopherson, D.G. (1946) Structural Defence Ministry of Home Security, Research and Experiment Department, R.C 450.

Christoforous, A.P., Yigit, A.S., Majeed, M. (2013). Low Impact Velocity Impact Response for Structures with Local Plastic Deformation: Characterization and Scallling. *Journal of Computational Nonlinear Dynamic,* 8: 011012-1.

Corbett, G.G. and Reid, S.R. (1993). Quasi-Static and dynamic local loading of monolithic simply-supported steel plates. *Int. J. impact Engng* 13: 432-441

Corbett, G.G., Reid, S.R. and Johnosn, W. (1996). The impact loading of plates by free flying projectiles: A review. *Int. J. Impact Engng* 2 (18): 141-230.

Corran, R.S.J., Shadbolt, P.J., Ruiz, C. (1983). Impact loading of plates - an experimental investigation. *International Journal of Impact Engineering,* 1 (1): 3-22.

Dallard, P.R.B. and Miles, J.C. (1984). In structural impact and crash worthiness. Marton, J. (ed) *Elsevier Applied Science* 2: 379-382.

De Rauvray, A., Arnaudeau, F., Duios, J., Chedmal, F.J and Haug, E. (1984). In Structural Impact and Crashworthiness. (Davies, G.A.O. ed) *Elsevier Applied Science* 1: 193- 242.

Dey, S., Borvik, T., Hopperstad, O.S., Leinum, JR., Langseht, M. (2004). The effect of target strenght on the perforation of steel plates uing three different nose shapes. *Int. J. Impact Eng.,*30: 1005-1038.

Dey, S., Borvik, T., Teng, X., Wierzbicki, T., Hopperstad, O.S. (2007). On the ballistic resistance of double-layered steel plates: An experimental and numerical Invesigation. *International Journal of Solids and Structures,* 6701-6723.

Duffel, T.A., Cheresh, M.C. and Surtherland, S.H. (1984). Experimental invesitigation of scalling laws for punch-impact loaded structures. *Int. J. Impact Engng.,* 2: 103-117.

Feyel, F. and Chaboche, J.L. (2000). FE^2 Multiscale approch for modelling the elastoviscoplastic behaviour of long fiber SiC/Ti composite materials. *Computer Methods in Applied Mechanics and Engineering,* 183(3-4): 309-330.

Goldsmith, W. and Finnegan, S.A. (1971). Penetration and perforation process in metal targets at and above ballistic velocities. *Int J. Mech Sci*. 13: 843-866.

Iqbal, M.A., Gupta, N.K . (2008). Energy absoption characteristuc of aluminum plates subject to projectile impact. *Latin American Journal of Solids and Structures,* 5 (4): 259-87.

Johnson, W. (1972). *Impact strength of materials*. London: Edward Arnold.

Jonas, G.H and Zukas, A.J. (1978). Mechanics of penetration: analysis and experiment. I*nt. J. Engng Sci.*, 16: 879-904.

Jones, N. (1993c). Recent Studies on the Response of Structures Subject to Large Impact Loads. *The Society of Naval and Architecture and Marine Engineers and The Ship Structure Committee Symposium '93*.

Jones, N. (1984). In structural impact and crash worthiness. Davies G.O (ed) *Elsevier Applied Science* 2: 369-382.

Jones, N., Birch, R.S., Duan, R. (2008). Low velocity perforation of mild steel rectangular plates with projectiles having different shaped impact faces. *Journal of Pressure Vessel Technology,* 130(3): 031206-1-031206-8.

Jones, N. and Paik, J.K. (2012). Impact Perforation of Steel Plates. *Ships and offshore structures.*

Kane, A., Borvik, T., Hopperstad, O.S., Langsethe, M. (2009). Finite Element Anlaysis of Plugging Faliure in Steel Plates Struck by Blnt Projectiles. *Journal of Applied Mechanics,* 76: 0513021- 1.

Kar, A.K. (1978a). Impact load for tornado generated missile *Nuclear Engineering and Design* 47 (1): 107-114.

Kar, A.K. (1978b). Loading time history for tornado generated missile. *Nuclear Engineering and Design* 141 (3): 375-385.

Karamnejad, A. and Sluys, L.J. (2014) Modeling Concrete Under High Frequency Loading Using a Multi-Scale Method. *Computational Modeling of Concrete Structures Taylor and Fanccies Group,* ISBN 978-1-138-00145-9.

Kouznetsova, V., Brekelmans, W. A. M. and Baaijens, F.P.T. (2001). An approach to micro-macro modelling of heterogeneous materials. *Compuatational Mechanics,* 27 (1): 37-48.

Levy, N. and Goldsmith, W. (1984). Normal impact and perforation for thin plates by hemispherically tipped projectile *Int J Impact Engng.,* 2: 209-229.

Levy, N. and Goldsmith, W. (1984). Normal impact and perforation for thin plates by hemispherically tipped projectile. *Int J Impact Engng.,* 2: 299-234.

Li, Q.M., Reid, S.R., Wen, H.M. and Telford, A.R. (2005). Local Impact Effects of Hard Missile on Concrete Targets. *International Journal of Impact Engineering* 32: 224-284.

Liang, C.C., Yang, M.F., Wu, P.W., Teng, T.L. (2005). Resistance of prformace of multi-layered targents using an estimation procedure with marine application. *Ocean Engineering,* 32(3- 4): 441-468.

Mohotti, D., Ali, M., Ngo, T., Lu, J., Mendis, P., Dong, R. (2013). Out-of-Plane Impact resistance of Aluminium Plates Subjected to Low Velocity Impacts. *Material and Design,* 50: 413-426.

Muncaster, R. (1993*) A level physics* (4th edition) Cheltenham: Nelson Thornes Ltd

Mughal, M.A., Smith, S.J. and Roberts, A.C. (1994). Design Guide for Structures Subject to Impulse and Impactive Loads.

Neilson, A.J. (1980). Missile Impact on Metal Structures. *Nuclear Energy* 19 (3): 191-198.

Neilson, A.J. (1983). Hard missile impact on plates and tubes. *Seminar on the State of the Art in the Simulation of Impulsive and Impact Loading St Peters College* Oxford.

Nelson, J.A. (1985). Empirical equations for the perforation of mild steel plates. *Int .J. Impact Engrg.,* 3: 137-142.

Ohte, S., Yoshizawa, H., Chiba, N. and Shida, S. (1982). Impact Strength of Steel Plates Struck by Projectiles. *Bull JSME* 25. 206: 1226-1231.

Paul, B. and Zaid, M. (1958). Normal perforation of a thin plate by truncated projectiles" *J Franklin nstitute* (256): 317-335.

Pradhan, P.K., Gupta, N.K., Ahmad, S., Biswas, P.P., Dayanand. (2017). Numerical Investigations of Spherical Projectile Impact on 4mm Thick Mild Steel Plate. *11th International Symposium on Plasticity and Impact Mechanics, Implast., :*109-115.

Recht, R.F. and Ipson, T.W. (1963). Ballistic perforation dynamics. *J. Applied. Mech* (30):384- 390.

Robinson, B. (1742). *The New Principles of Gunnery*.

Rosenbery, Z. and Dekel, E. 2016. *Terminal Balistics.* 2nd Edition ed. Singapore: Springer.

Shadi, B., Owais, S., Mohammad, A. (2015). Similitude and Scaling Laws - Static and dynamic behaviour of beams and plates. *1st International Conference on Structural Integrity..*

Shen, W.Q and Jones, N. (1991a). Mechanics of Structures and Machines. 19 (4) :PP.527-549.

Shen, W.Q and Jones, N. (1993b). *Int. J. Impact Engng.,* 13 (4)

Shen, W.Q and Jones, N. (1993). Dynamic Plastic Response and Failure of Clamped Beam Struck Transversely by a Mass. *Int. J. Solids Structures* 30 (12): 1631-1648.

Sonin, A., (2001). *The Physical Basis of Dimensional Analysis.* Cambridge M.A

Stronge, W.J. In metal forming and impact mechanics. Reid, S.R. (2016) (ed) Pentagon press: PP 289-302.

Stronge, W.J. (2018). *Impact Mechanics.* First ed. Cambridge United Kingdom: Cambridge University Press.

Srivathsa, B., Ramakrishnan N. (1997). On the ballistic performance of metalic materials. *Bull. Mater. Sci.,* 1 (20): 111-123.

Taylor, G.I. (1948). The Formation and enlargement of a circular hole in a thin plastic sheet. *Quart .J. Mechanical Appl. Math. 1*: 103-124.

Taylor G.I. (1948). The formation and enlargement of circular hole in a thin plastic sheet. *Quart .J. Mech Appl. Math. 1*: 103-124.

Thompson, W.T. (1955). An approximate theory of armour penetration. *J. Appl. Phys.* 26 (1): 80-82.

Voyiadjis, G. Z., Deliktas, B., Palazotlo, N.A. (2009). Thermodynamically consistent coupled viscoplastic damage model for perforation and penetration in metal matrix composite material. *Elseiver,* part B (40): 427-433.

Voyiadijis, G., (2013). "Coupled Viscoplastic Damage Model for Hypervelocity Impact Induced Damage in Metals and Composites". *Material under Extreme Loading: Application to Penetration and Impact.*

Warren, T.L. and Poorman, K.L., (2001). Penetration of 6061-T6511 aluminum targets by Orgive-nosed VAR 4340 Steel Projectiels at Oblique Angles Experiements and Simulations. *International Journal of Impact Engineering,* 25:. 993-1022.

Wessman, W.E. and Rose, W.A. (1942). *Aerial Bombardment Protection*. America.

Wen, M-H. and Jones, N. (1992). Semi-empirical equation for the perforation of plates struck by a mass. *In structures under shock and impact.* Bulson, P.S (ed) *Computational mechanics publication*: 368-380.

Wen, W.H. and Reid, S.R. (1998). Deformation and Perforation of cylindrical shells struck normally by blunt projectiles *International Journal of Pressure vessel and piping* (75): 213- 219.

Wen, H. (2000). Predictin the Penetration and Perforation of FRP Laminates Struck Normally by Projectiels with Different Nose Shapes. *Composite Structures,* 49(3): 321-327.

Zaid, M. and Paul, B. (1957). Mechanics of high- speed projectile perforation" *J. Franklin Institute* (256): 317-335.

Zukas, A.J., Scheffler, D.R. (2000). Practical aspect of numerical simulations of dynamic events: effects of meshing. *Army Research Laboratory.*