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Physics-Informed Neural Networks for the Prediction of Critical Sand Transport Velocity in Oil and Gas Pipelines

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Abstract
Poorly consolidated reservoir formations make the generation of sand alongside hydrocarbons unavoidable. The condition of petroleum pipelines can be seriously compromised by the rapid generation of sand or by sand deposition at low velocities, leading to degradation and reduced capacity. Consequently, it is essential to investigate the minimum transport velocity needed to prevent pipeline sand accumulation. This study employed two predictive modelling tools, a physics-informed neural network (PINN) and a multilayer perceptron (MLP) regressor, to estimate the minimum transport conditions in a solid-liquid-gas pipeline. The models were developed using 182 experimental data sets, which included variables such as superficial velocity, pipe diameter, particle diameter, sand density, sand concentration, liquid density, viscosity, pipe angle of inclination, and critical velocity parameters. The results indicated that the physics-informed neural network outperformed the MLP regressor, achieving an R^2 coefficient of 0.9999 and a root mean square error (RMSE) of 0.00465. In comparison, the MLP regressor attained an R^2 coefficient of 0.9992 and an RMSE of 0.0295. Both models were evaluated alongside existing empirical and data-driven models and demonstrated superior performance in predicting minimum transport velocity, with the PINN yielding the best results. A sensitivity analysis revealed that the superficial velocity is the most influential parameter for predicting minimum transport velocity, followed by pipe diameter, sand concentration, and pipe angle. This research highlights the potential of effectively integrating physical laws into the machine learning training process to estimate minimum transport velocity in multiphase pipelines better

Keywords: Minimum Transport Condition, Minimum Transport Velocity, Physics Informed Neural Networks, Multiphase System

1.0 INTRODUCTION

Trace amounts of solids such as sand and debris, sometimes get transported with fluids from a reservoir due to fractures in loosely packed formations and poorly consolidated reservoirs. This flow of sand and oil through pipelines creates significant flow assurance challenge. Sand deposition may block pipelines and disrupt oil and gas transport. Furthermore, sand particles may be responsible for the pipe abrasive wear, erosion, and even corrosion of the pipe surface, either by mechanical action or microbial activity under sand layers. All these effects weaken pipeline integrity and reduce the efficiency of production and transport. There is the need to ensure uninterrupted flow of hydrocarbons from the reservoir to the processing facilities.

Sand production happens when formation stress surpasses formation strength and leads to rock failure, which occurs due to geological movements, ground stress, pressure-induced, and flow resistance force. Sand particle production can consist of load-bearing solids and formation fines (Mahmud *et al*., 2020). If the production of sand is likely to occur, sand penetration into the system can be avoided by the use of some methodologies, including the subsurface sand rejection mechanism such as gravel packs and screens; however, these methods may cause a major decline in productivity (Leporini *et al.*, 2019). Sand particles need to be transported to prevent settling in pipelines when transporting hydrocarbons. Thus, the flow rate must be kept above the minimum transport conditions (MTC) or minimum transport velocity (MTV). This ensures sand particles keep moving along the pipe. Experts use different ways to estimate this velocity, including mechanistic models, Computational Fluid

Dynamics (CFD) models, and experimental studies. Several studies have focused on developing models to estimate minimum transport velocity. Al-Mutahar (2006) proposed a mathematical model for the critical transport velocity based on the equilibrium condition under which the net force equals zero, using the two irregular flow theories of Oroskar and Turian (1980) and Davies (1987). In developing this mathematical model, Al-Mutahar employed a three steps approach. The model first calculates the turbulent velocity changes that are necessary to keep solids in suspension, then estimates the actual turbulent changes caused by the moving fluid and finally determines the critical transport velocity. This model assumes that when the required and produced turbulent changes are equal, the solids remain in suspension.

Tebowei *et al*. (2018) presented a 3D CFD model that simulated fluid behaviour in the Eulerian-Eulerian process with the dynamic principle of particle flux to investigate the transportation of sand in turbulent pipelines and flow lines. The model considered inter-particle collision and friction forces to capture various regimes of particle movement. Results indicated that the inclined pipe affected the minimum transport velocity and properties of sand. The model considered that the use of correlations (empirical or semi-empirical) to determine the minimum sand transport velocity from inclined pipes, including horizontal pipes, could be unreliable for the inclined sections. Unlike the case of horizontal pipes, the V-inclined curve needed a far larger threshold velocity to keep the sand in suspension within the liquid.

Archibong-Eso *et al.* (2020) studied sand transport in horizontal and inclined pipes under Minimum Transport Condition (MTC) requirements, using experiments with pipes of 0.0127 m diameter and sand concentrations from 0.1% to 10%. The results showed that increasing mixture velocity or sand concentration raised both MTC and pressure gradients, stabilizing the pressure gradient at higher sand concentrations near MTC.

Several studies have explored data-driven approaches, such as machine learning, to model minimum transport conditions (MTC). Bhattacharya et al. (2007) simulated sand transport using experimental data, applying model trees and artificial neural networks (ANNs). Salam *et al.* (2018) used response surface methodology to create a regression model for predicting MTC based on a double-factor synergy, which they validated and refined. An improved approach by Sarraf Shirazi and Frigaard (2021) combined ANN and support vector machines (SVM) to predict the minimum velocity for transitioning from laminar to turbulent slurry flow, also calculating frictional pressure drops in pipes. Ehinmowo *et al.* (2021) assessed ANN, adaptive neuro-fuzzy inference system (ANFIS), and response surface methodology models for predicting MTC in multiphase pipelines, finding them superior to earlier methods. Further improvements by Ehinmowo *et al.* (2022) involved applying the firefly optimization algorithm to enhance a model for predicting the minimum velocity needed to keep sand particles suspended in multiphase flow, achieving over 98% accuracy, a 17% improvement over the benchmark. Stachurska *et al.* (2022) introduced new machine learning techniques to predict the velocity of loose tidal grains over dunes using particle visual velocity measurements. They applied the Student Psychology-based Optimization (SPBO) algorithm, which included linear, nonlinear, and exponential regression methods as the training models, and the Classification and Regression Tree (CART) and ANFIS.

Most studies have identified viscosity, size, density, and velocity as the factors affecting MTC for sands and focused on horizontal or nearly horizontal flows, with very few on vertical, curved, or inclined ones. Machine learning opens up new avenues to address some of these challenges. Previous machine learning implementations for MTC have outperformed the traditional approaches many times. However, while training, most models hardly consider physical laws that may enhance the precision of a model, its generalization capability, and adherence to MTC principles. This gap necessitates this study, which developed a Physics Informed Neural Network and a Multilayer Perceptron Regressor using experimentally obtained data in modeling minimum transport velocity.

2.0 MATERIALS AND METHOD

This study aims to predict the minimum transport velocity in multiphase flow systems based on viscosity, fluid and gas density, pipeline diameter, inclination angle, sand concentration, and sand particle size. Two nonlinear machine learning methods, the physics-informed neural networks (PINN) and a multilayer perceptron (MLP) regressor, were employed. The minimum transport velocity is the speed needed to keep sand in continuous motion, making this a regression problem since it involves predicting a continuous variable.

2.1 *Machine Learning and Algorithm used*

Physics Informed Neural Networks (PINNs): Physics-informed neural networks are a class of neural networks that solve problems in which data and physical laws are essential. PINNs incorporate known physical equations into their training process, **unlike regular neural networks**. A typical PINN network is shown in Figure 1.

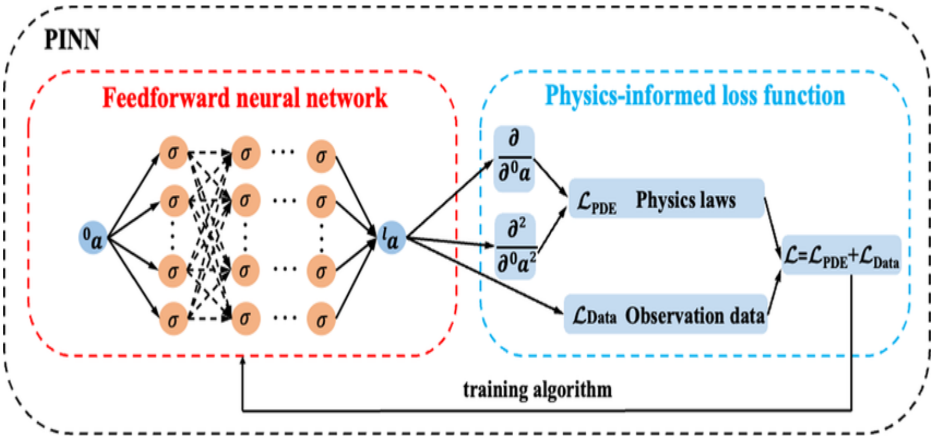

Figure 1: Physics informed neural network diagram (Li *et al.*, 2024)

The Turian *et al.* (1987) equation was implemented in this study to incorporate a physics-based model for the critical velocity in pipelines, which it uses as a guiding principle for training the neural network. It factors in pipe diameter, particle size, sand concentration, and fluid viscosity to estimate transport velocity. The Turian equation is incorporated into the loss function of the PINN modeling process. This addition helps the model learn the patterns in the data and the principles that govern the Turian *et al.* (1987) equation as shown in equation 1.

$$V_c = 1.7951\, C_v^{0.109} (1 - C_v)^{0.25} \left[\frac{D\, \rho_l \sqrt{gD(s-1)}}{\mu}\right]^{0.018} \left(\frac{d}{D}\right)^{0.06623} \sqrt{2gD(s-1)} \quad (1)$$

Where V_c is the critical transport velocity, μ is liquid viscosity, C_v is sand concentration, d is particle diameter, g is gravitational acceleration, D is pipe diameter, ρ_l is liquid density, and *s* is the ratio of particle density to liquid density

Multilayer Perceptron (MLP) Regressor: The major components of a multilayer perceptron include an input layer, several hidden layers, and an output layer as shown in Figure 2. Neurons form layers and are completely connected between layers, meaning that every neuron in a particular layer is linked to every neuron in the layer directly past it. An activation function is assigned to all neurons except those from the input layer. This activation function decides the neuron output based on the input provided. MLP is trained using a backpropagation method of adjustment in the neuron connections so that the training error is minimized and thus increases the accuracy.

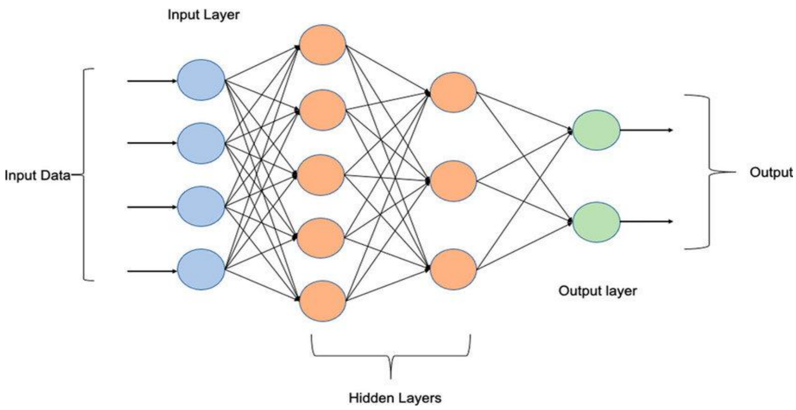

Figure 2: MLP Regressor Schematics (Afan et al., 2021)

2.2 Model Evaluation Metrics

For this study, mean absolute error (MAE), mean squared error (MSE), root mean squared error (RMSE), mean absolute percentage error (MAPE), and regression coefficient (R^2) were used to evaluate the performance of the models developed.

1. Coefficient of Determination or Performance (R-Squared)

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(V_{MTC,\ actual} - V_{MTC,\ prediction})^2}{\sum_{i=1}^{n}(V_{MTC,\ actual} - V_{MTC,\ mean})^2} \quad (2)$$

Where,

$V_{MTC,\ actual}$ = experiment minimum transport velocity values
$V_{MTC,\ predict}$ = predicted minimum transport velocity values
n = number of records
$V_{MTC,\ mean}$ = mean of minimum transport velocity values

2. Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(V_{MTC,\ actual} - V_{MTC,\ prediction})^2}{n}} \quad (3)$$

3. Mean Squared Error (MSE)

$$MSE = \frac{\sum_{i=1}^{n}(V_{MTC,\ actual} - V_{MTC,\ prediction})^2}{n} \quad (4)$$

4. Mean Absolute Error (MAE)

$$MAE = \frac{\sum_{i=1}^{n} |\ V_{MTC,\ actual} - V_{MTC,\ prediction}\ |^2}{n} \tag{5}$$

5. Mean Absolute Percentage Error (MAPE)

$$MAPE = \left|\frac{V_{MTC,\ actual} - V_{MTC,\ prediction}}{V_{MTC,\ actual}}\right| * 100 \tag{6}$$

2.3 Data Collection and Preparation

This study uses data from Archibong-Eso *et al.* (2020) experimental work. The experiments were conducted at the Oil and Gas Engineering Centre of Cranfield University using a 0.0254-meter diameter pipe placed horizontally and at a 30-degree incline. Table 1 summarizes the data.

Table 1: Summary of statistics of the experimental data used in this study

Features	Minimum	Maximum	Mean	Median	Standard Deviation
Superficial Velocity (ms^{-1})	0.07	8.37	2.12	1.52	1.81
Pipe Diameter (m)	0.009	0.70	0.100	0.100	0.091
Particle Diameter (mm)	0.10	3.70	0.37	0.36	0.37
Sand Density (kgm^{-3})	2650	2650	2650	2650	0
Sand Concentration (vol %)	0.002	60.00	12.07	10.00	14.27
Liquid Density (kgm^{-3})	850.0	1000.0	997.9	1000.0	16.6
Liquid Viscosity (mPa·s)	1.00	17.00	1.15	1.00	1.37
Pipe Angle (ø)	-25.00	30.00	4.54	0.00	11.39
Critical Velocity (ms^{-1})	0.07	4.40	1.64	1.44	1.01

One hundred eighty-two (182) data points were acquired for superficial velocity, pipe diameter, particle size, sand concentration, liquid density and viscosity, and pipe angle. These experiments were carried out using video recordings and pressure sensors to track sand transport through the liquid stream, focusing on minimum transport conditions. Before developing the model, the data was cleaned and scaled. It was then split into three sets: 70% for training, 15% for validation, and 15% for testing.

2.4 MODEL ARCHITECTURE FOR TRAINING

The PINN has a structure of 8-72-72-1. This means it has 8 input nodes (one for each feature), two hidden layers with 72 nodes each using the tanh activation function, and 1 output node for predicting critical velocity. It was trained with a learning rate of 0.001 for 500 epochs and includes physics-based rules for better understanding. The MLP Regressor follows an 8-50-50-1 architecture. It starts with 8 input nodes, has two hidden layers with 50 nodes each using the ReLU activation function, and 1 output node for predicting critical velocity. This model is trained with a learning rate of 0.001 for 600 epochs. Both models use deep learning to find complex patterns in the data, with the PINN adding some physics rules to make predictions more reliable.

3.0 RESULTS AND DISCUSSION

A model evaluation study was done after modeling to test the performance of PINN and MLP models in the test or future scenarios. Table 2 presents the performance of each model evaluated in terms of MAE, MSE, RMSE, MAPE, and R^2 metrics. The PINN had superior prediction in comparison with the MLP regressor. For PINN and MLP models, R^2 and RMSE values were equal to 0.9999 and 0.9992 and 0.0047 and 0.03, respectively.

Table 2: Performance evaluation of PINN and MLP Regressor

	R^2	MSE	RMSE	MAE	MAPE (%)
PINN Model	0.99998	0.00002	0.00469	0.00367	0.24904
MLP Model	0.99920	0.00087	0.02950	0.01974	1.54660

3.1 Model Evaluation Visualization

Figure 3(a) and 3(b) show prediction error plots for the PINN and MLP models respectively. A prediction error plot is a plot of the expected values of MTC from the dataset against the MTC values predicted by the model. The plot helps determine how well the predictions correspond to the actual values.

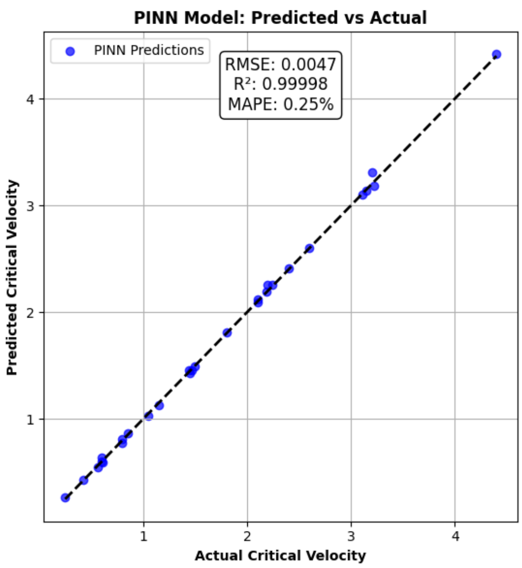

Figure 3(a): PINN model prediction error plot

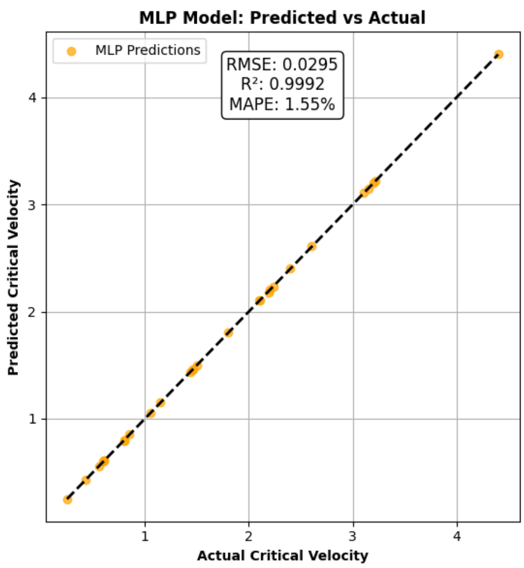

Figure 3(b): MLP model prediction error plot

3.2 Comparative Analysis with existing MTC Models

Table 3 compares the developed model's results with those of other published models. The comparison shows that the PINN and MLP regressor models performed better than the other literature models using the R^2, MSE, RMSE, and MAPE metrics.

Figure 4 shows the visualization of the sensitivity analysis conducted to evaluate how different factors affect the minimum transport velocity of sand using the PINN model. The results showed that superficial velocity has a strong positive correlation with the sand's critical transport velocity. As superficial velocity increases, the critical velocity of the fluid rises as well. This is because higher fluid velocities would exert more drag on sand particles, thus providing the required energy to keep them suspended and not deposited on pipeline walls (Dabirian *et al.*, 2016; Yao *et al.,* 2022). The pipe diameter also has a positive relationship with the sand's critical transport velocity. When the pipe gets bigger, you need a higher critical velocity to keep sand particles floating in the fluid. This happens because, with the same flow rate fluid moves slower in bigger pipes due to more area inside. To prevent sand from settling, the critical velocity must be higher to ensure suspension in these larger pipes (Yao *et al.,* 2023). Although particle diameter shows a slight positive trend, its effect on critical velocity is relatively weak.

Table 3: Numerical comparison between the developed models and published models

Correlation	R^2	MSE	RMSE	MAPE (%)
Turian et al. (1987)	0.6932	0.3315	0.5758	27.4
Danielson (2007)	0.8003	0.17448	0.4170	60.3
Yan (2010)	0.803	0.07129	0.2670	26.2
Fajemidupe et al. (2019)	0.8149	0.00090	0.0300	24.0
Ehinmowo et al (2021)	0.9994	0.00208	0.0456	2.15
Ehinmowo et al (2022)	0.9845	4.99×10^{-17}	7.07×10^{-9}	22.97
PINN Model	0.99998	0.00002	0.0047	0.2
MLP Model	0.99920	0.00087	0.0295	1.5

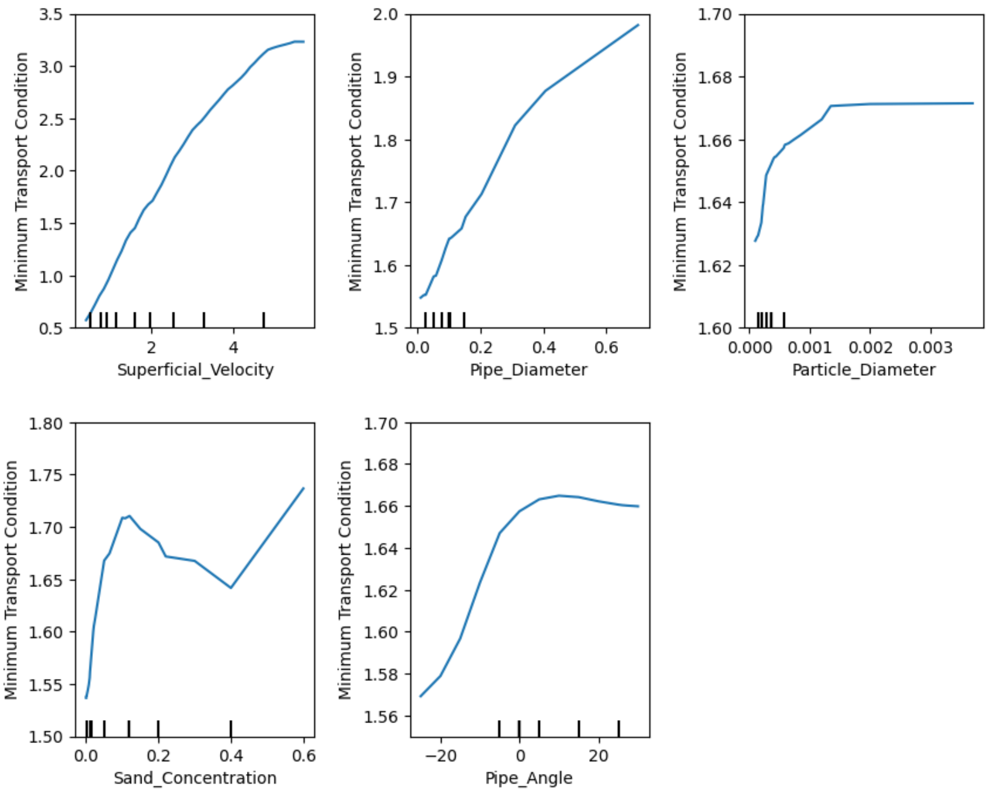

Figure 4: Sensitivity analysis for minimum transport condition prediction

The relationship between sand concentration and critical velocity is non-linear, with noticeable changes as concentration increases. An increase in sand concentration results in a greater degree of particle-particle interactions, which in turn can cause an increase in the effective viscosity of the fluid, thus a greater critical velocity is to be used in order to prevent deposition (Zhang *et al.,* 2022). The pipe's inclination angle also has a non-linear effect on critical velocity. For upward-inclined pipes, the flow has to work against gravity; hence, higher velocities are required in order to prevent sand from settling. In contrast, gravitational effects become smaller for horizontal or downward-sloping pipes, and thus a lower critical velocity is needed to keep particles in suspension (Vlasak *et al.,* 2019). However, beyond a certain angle, the effect of the pipe's inclination angle becomes negligible, showing diminished effects of gravity.

This analysis underscores that superficial velocity is the most influential parameter for minimum transport velocity modelling, followed by pipe diameter, sand concentration, and pipe angle. The particle diameter has the least effect. The intricate interdependence of these parameters, which must all be considered to fully grasp the conditions of minimum sand transport, adds a layer of complexity to the problem.

4. CONCLUSION

Physics-informed neural networks and multilayer perceptron regressors were developed in this study to predict the minimum transport velocity that will keep sand particles in motion in multiphase solid-liquid-gas flow lines and pipelines. The models correlate the critical velocity in terms of sand particle concentration, diameter of the pipe, superficial velocity, inclination angle, and diameter of the sand particle. The physics-informed neural networks showed a better prediction capability than the MLP Regressor, with an R2 score of 0.9999 and RMSE of 0.00465. The MLP Regressor also performed well with R2 scores of 0.9992 and RMSE of 0.0295, respectively. The developed models were compared with existing empirical and data-driven models. From the outcome, the two models performed better in predicting the minimum transport velocity with PINN performing best. This research has the potential to significantly impact the field of fluid dynamics and machine learning applications, particularly in the development of machine learning models that incorporate physical laws to indicate the minimum transport condition. Additionally, the potential for more accurate physical laws to be investigated with more experimental data and parameters collated for more robust modelling of the minimum transport condition is a promising avenue for future study, further highlighting the significance of the work.

NOMENCLATURE

Symbol	Definition
MTC	Minimum Transport Condition
PINN	Physics-Informed Neural Network
MLP	Multilayer Perceptron Regressor
CFD	Computational Fluid Dynamics
ANN	Artificial Neural Network
SVR	Support Vector Regression
ANFIS	Adaptive Neuro-fuzzy Inference System
RSM	Response Surface Methodology
RMSE	Root Mean Squared Error
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
R^2	Coefficient of Determination

FFA	Firefly Optimization Algorithm
SPBO	Student Psychology-Based Optimization
SVM	Support Vector Machine
CART	Classification and Regression Tree
KAN	Kolmogorov Arnold Networks
PG	Pressure Gradient

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